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A hybrid inventory policy with split delivery under regular and surge demand

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ABSTRACT

This paper proposes a hybrid inventory policy with split delivery under regular and surge demand. The combination of regular and surge demand can be observed in many areas, such as healthcare inventory and humanitarian supply chain management. The arrival rate of regular demand is typically higher than the arrival rate of surge demand, whereas the volume of regular demand is typically lower than the volume of surge demand. This paper proposes an inventory management model that considers both emergency and regular replenishments corresponding to both demand patterns. The equilibrium equations developed for this model are based on the level crossing theory. These equations are used to develop a search-based heuristics to identify near optimal inventory management policies. Numerical results show that the proposed hybrid inventory policy with split delivery outperforms similar hybrid inventory policy without split delivery when holding and shortage costs are relatively low.

1. Introduction

Each year, millions of people around the globe are affected by man-made and natural disasters, such as terrorist attacks, earthquakes, disease outbreaks, volcanic eruptions, floods, tsunamis, storms, and hurricanes. These disasters trigger a surge demand for humanitarian products such as, medication, food, water, etc. Consequently, hospitals, retailer stores and manufacturers face a surge demand for their inventories. Having the necessary resources to respond to surge demand orders in a timely manner is important because a shortage of these supplies impacts people's lives. Shortages of supplies also have financial impacts. United Nations International Strategy for Disaster Reduction Secretariat (UNISDR) (2012) estimates that in 2011 the global economic losses due to disasters ranged between $350 billion and $380 billion. Within a small period of time, the probability that a particular facility faces surge demand due to a disaster is very low. However, the size of a surge demand is typically very large. Providing humanitarian relief on a timely manner to victims could turn into a major challenge if hospitals, retailers, and manufacturers have not planned well in advance. For example, the lack of a well-coordinated relief plan during the earthquake in Haiti delayed the delivery and distribution of the necessary supplies (Beresford and Pettit, 2012). Similarly, in 2005, Hurricane Katrina greatly impacted the economy of a number of southern states in the US. Many manufacturers and retailers ran out of stock of certain emergency items due to an overwhelmingly high demand which they were not prepared to handle. One lesson learned from these experiences is that being proactive, by developing a disaster management plan, has the potential to save lives, reduce financial loss, protect business assets, help businesses recover faster, and provide greater security and control in case of a disaster (Belson, 2005). Developing a disaster management plan is very important for healthcare providers and their suppliers because many types of medicines are critical to saving lives. Thus, healthcare providers are pressured to carry inventories of medicines that are needed in case of a disaster. Some of these inventories are very expensive and often have a short shelf-life, such as blood and medicines. Identifying policies that can help to manage these inventories is crucial to responding on-time to disasters, and therefore saving lives and money.

Managing the inventory of products under surge demand could be challenging for both humanitarian services (e.g. Beamon and Kotleba, 2006; Balcik and Beamon, 2008) and the manufacturing industry (e.g. Hendricks and Singhal, 2003, 2005). In traditional manufacturing, if the demand forecasts are inaccurate or the order replenishment does not follow the plan, it leads to delayed orders or at worst canceled orders. However, in healthcare delivery (esp.

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hospitals) or humanitarian services, a shortage could make the difference between life and death. Therefore, healthcare providers often maintain high service levels for life-saving items. This is achieved by either increasing the inventory level or through expedited shipments. Both ways are expensive. In order to minimize the inventory holding cost, while maintaining high service levels, healthcare providers and their suppliers need to develop more sophisticated and proactive inventory management policies for items that may face surge demand. Motivated by the challenges of meeting surge demand, this paper proposes a hybrid inventory model under regular and surge demand. This paper incorporates the concept of split delivery in a hybrid inventory policy. Split delivery could be an effective way of reducing inventories while maintaining acceptable service levels. Split delivery considers the scenario in which there is a single supplier who replenishes all orders but the supplier could split the delivery of the orders with multiple shipments. By using split delivery, a service provider can hedge against the risk of possible long lead-times when the lead-times of the suppliers are stochastic. As split delivery has the potential to reduce the long lead-times, using this procedure a service provider is able to reduce the safety stock needed to maintain high service targets.

The stochastic lead time of a standard delivery may cause longer time to fulfill a replenishment order in an inventory system facing regular and surge demand. An inventory model with standard delivery and stochastic lead time may cause longer lead time and requires excess inventory to avoid shortage, especially when the required service level is high. Alternatively, an inventory system that allows split delivery can reduce the dependency on single deliveries and allows orders to arrive in a different time frame. A single replenishment order is broken into multiple replenishment orders, therefore avoiding possible long lead times. Splitting the delivery of a shipment has the potential to reduce the safety stock required to maintain high service targets. As a result, split delivery of regular order conjunction with emergency order allows reduction in the safety stock and realizes savings in the holding and the shortage costs at the expense of a possible increase in ordering cost. Cost tradeoff between split and standard deliveries depends on various input parameters such as unit holding cost, shortage cost, delivery lead time, ordering cost, regular demand arrival rate etc. It can be shown that considerable cost savings can be obtained from split delivery depending on various instances. Moreover, the existence of surge demand forces a service provider to maintain high levels of safety stock in order to provide high service levels. Specially, when surge demand has a high arrival rate with a large demand volume, an inventory system that relies on standard delivery would require excess inventory to avoid shortage. As split delivery allows orders to arrive in a different time frame, it allows adjusting the safety stock more frequently. This results in reduction of the safety stock, and savings in the holding and the shortage costs.

Motivated by the challenges of surge demand and by the knowledge that split delivery of orders is an effective way of reducing inventories, this paper considers a continuous-review inventory system for a single product that faces regular and surge demands and allows multiple outstanding orders. The goal is to identify an inventory management policy that minimizes cost while maintaining a high service level. The inventory policy identifies two reorder points, regular order point and emergency order point, and two order quantities, regular order quantity and emergency order quantity. We propose a mathematical model that is based on the level crossing theory (LCT) in order to capture the delivery split. Furthermore, we develop a Tabu search-based algorithm to find high quality solutions in a reasonable amount of time.

This paper uses LCT to obtain the long-run average cost under a given policy. Applying LCT to obtain equilibrium distribution has advantages over other methods such as the Markov decision process and queuing theory. The state-action space in the underlying semi-Markov decision process is large for the problem addressed in this paper. The queuing theory requires extensive, tedious and time consuming analysis to derive integral equations from the Lindley recursion in order to obtain the probability density function in complex stochastic models with state dependencies . LCT is a faster and easier method to derive equations for the probability density. The classical renewal theory is often used to model a variety of continuous review inventory problems that consider the random demand process as a compound Poisson process with a general distribution for demand sizes (Brill, 2000; Beamon and Kotleba, 2006). Under an assumed order policy, such as (s, S) or (R, Q), the analysis of those studies relied on the assumption that back orders or lost sales are triggered only during the replenishment lead time. This paper allows back orders or emergency orders to be triggered at any point in time. Therefore, the classical renewal theory is not an option to obtain the equilibrium distribution.

In summary, this paper contributes to the existing literature in the following ways: (a) it incorporates the concept of delivery splitting into the analysis of a continuous review, stochastic inventory model under regular and surge demand; (b) it uses LCT to derive a stationary distribution function of the inventory level for a continuous review inventory system characterized by (i) regular and surge demand, (ii) split delivery, (iii) variable lead time, two reorder points, and two types of orders, and (iv) shortage cost; (c) it considers inventory levels to be discrete states while most of the papers considered the inventory level as continuous states during applying LCT.

The remainder of this paper is organized as follows. Section 2 provides a review of the relevant literature in the area of the inventory management and the humanitarian supply chain management. Section 3 discusses the modeling approach based on the level crossing theory. Section 4 presents a Tabu search-based heuristics to solve the problem. Section 5 presents the results of the numerical analysis. This section also tests the performance of the Tabu search heuristics and proposed hybrid inventory policy under various scenarios. Finally, Section 6 summarizes the findings and concludes the paper.

2. Relevant literature

Studies on optimizing the inventory system of emergency items in support of humanitarian operations have mostly been focused on strategic and operational planning. For example, problems which coordinate facility location and inventory planning decisions under emergency situations have been studied by a number of researchers, such as Beamon and Kotleba (2006), Chang et al. (2007), Balcik and Beamon (2008), Lodree and Taskin (2008), Rottkemper et al. (2011), Ozguven and Ozbay (2012), Rawls and Turnquist (2010), Campbell and Jones (2011), Jin and Roni (2011), Jin et al. (2012), Roni and Jin (2013), etc. Stochastic inventory control with two types of demand classes has been studied for decades. A large body of literature has studied this problem in different settings. For example, Wang et al. (2013) studied the rationing policy in an inventory system with two demand classes and different service criteria for backorders with a continuous review (R, Q) system; Koçağa (2007) studied a single-echelon spare parts distribution system with two demand classes and proposed a static rationing policy that would ration stock to the non-critical class. Other researchers that studied continuous review stochastic inventory models with two types of demand are

Isotupa (2011, 2013), Fadıloğlu and Bulut (2010), Arslan et al. (2007), Benjaafar et al. (2010), and Deshpande et al. (2003). Inventory control models with two types of demand classes under periodic review have been studied by Wang et al. (2013), Möllering and Thonemann (2010), Chew et al. (2013), Hung et al. (2012), Frank et al. (2003), and Pourakbar and Dekker (2012). The mixture of demand processes have been studied by Azoury et al. (2012), Presman and Sethi (2006), and Sobel and Zhang (2001). The hybrid inventory management policy with two reorder points and two order quantities with split delivery has been the subject for a number of studies in the literature. Some of the studies related to the split delivery problem have been conducted by Janssen et al. (2000), Ching-Jong and Wen-Hwa (1994), Mohebbi and Posner (2002), Moinzadeh and Lee (1989), Glock and Ries (2013), and Janssen et al. (2000) showed the advantage of delivery splitting from the point supplier's point of view. Ching-Jong and Wen-Hwa (1994) considered an inventory system in which the replenishment of more than one item is triggered whenever the inventory level drops to the order point or lower, and they developed a procedure that minimizes the sum of the expected holding and shortage costs. Mohebbi and Posner (2002) developed a continuous-review inventory system with lost sales, non-unit-sized demand, multiple replenishment orders outstanding, and split deliveries. Moinzadeh and Lee (1989) considered an inventory system in which orders might arrive in two shipments and presented the operating characteristics and an approximate cost function for such a system. Glock and Ries (2013) presented mathematical models for a multiple-supplier single-buyer integrated inventory problem with stochastic demand and variable lead time and studied the impact of the delivery structure on the risk of incurring a stockout during lead time.

The hybrid inventory management policy with two reorder points and two order quantities has been studied by Moinzadeh and Nahmias (1988), Mohebbi and Posner (1999), and Roni et al. (2015). The study by Moinzadeh and Nahmias (1988) analyzes the inventory management policy of a continuous review system. The model proposed in this study considered four decision variables: regular reorder level, emergency reorder level, regular order quantity, and emergency order quantity. The model assumed: full backorders, constant but different lead times for the two types (regular and emergency) of orders, and at most one outstanding (regular and emergency) order. The authors derived an approximate expression for the average cost, and presented a heuristic procedure to determine the value of the decision variables. Mohebbi and Posner (1999) developed a model to manage the inventory for products with non-unit-sized demands. This paper also proposed a four-parameter inventory management policy to determine the following: regular order level, emergency order level, regular reorder point, and emergency reorder point. The paper assumed compound Poisson demand, at most one outstanding (regular and emergency) order, and exponentially distributed lead time for both orders. The study gave an explicit expression of the average cost function. Roni et al (2015) proposed a hybrid policy for a stochastic inventory system facing regular demand and surge demand with two reorder points and two order quantities. The following is a list of some of the continuous review inventory models for managing emergency orders: Johansen and Thorstenson (1998), Axsäter (2007), Kalpakam and Sapna (1994), Huang et al. (2011), and Mamani and Moinzadeh (2014). These studies are closely related to the work presented in this paper. Other studies that focus on periodic review stochastic inventory models with regular order mode and expedite order mode are Sheopuri et al. (2010), Jain et al. (2011), Arts et al. (2011), Cheaitou and Delft (2013), and Zhang et al. (2012).

This paper uses LCT proposed by Brill and Posner (1977) in order to derive the distribution of the inventory level at the equilibrium, under a given inventory management policy. Several researchers have used LCT in stochastic inventory systems in order to derive the stationary probability distribution of inventory level in continuous review models. For example, Brill and Chaouch (1995) used LCT to derive the distribution function and the corresponding expected value of on-hand inventory, size and frequency of orders, and total cost for different inventory management policies. Mohebbi and Posner (1998) applied LCT to derive the distribution of the on-hand inventory in a continuous-review inventory management system with: compound Poisson demand; Erlang, and hyper-exponentially distributed lead times; and lost sales. Mohebbi (2003) used LCT to compute the stationary distribution of the on-hand inventory in a continuous review system. The paper considered supply interruptions in which a supplier can assume one of the two states available or unavailable at any point in time. The system was modeled using a continuous-time Markov chain. Mohebbi (2006) used LCT to derive the stationary distribution of the inventory level in a limited capacity production-storage system with lost sales, stochastic piecewise linear production system, and compound Poisson demands. Chaouch (2007) used LCT to derive the long term inventory distribution function and determine replenishment strategies. The paper considered the scenario when buyers are faced with price-discounting campaigns which are random. As demonstrated above, most of the existing literature considered the inventory level to be in continuous states. Instead, this paper considers the inventory levels as discrete states of the system, and uses LCT to obtain the equilibrium distributions of inventory levels under a given inventory policy.

This work contributes to the existing literature related to stochastic inventory system facing regular demand and surge demand. The modeling, solution approach and analysis in this paper are unique in that there are no hybrid stochastic inventory models in the literature characterized by (1) regular and surge demand, (2) split delivery, (3) variable lead time, (4) two reorder points and two types of order, and (5) shortage cost. Moreover, we are not aware of any stochastic inventory model that considers the inventory level as discrete states while applying LCT. The model we propose is inspired by the challenges faced by healthcare providers or humanitarian logistics providers in managing their emergency supplies in responding to surge demands.

3. Model formulation

Consider a single item inventory management system facing two types of demands, regular demand and surge demand. The regular demand follows a Poisson process with an arrival rate equal to λ_1. The size of demand is one unit. The surge demand follows a compound Poisson process with an arrival rate of λ_2. The demand size follows a discrete distribution. The minimum demand size is a, maximum size is b, and r_k is the probability that demand is equal to k ($k \in [a, b]$). In reality, the size of surge demand is random and its range is wide. However, for sake of simplicity we assume that surge demand takes values within the interval $[a, b]$. In order to address this fact, we develop a discrete distribution function given by Eq. (12) in Section 5.1. Because of the nature of the two demand types, we assume λ_1 is larger than λ_2 , and $a > 1$.

The inventory management policy is defined by the following four decision variables: R denotes the reordering point for regular order, Q denotes the regular order quantity, R_e denotes the reordering point for emergency order, and Q_e denotes the order quantity for each emergency order. When the inventory position (stock on-hand + stock on-order) is equal to or decreases below reorder point R, an order of size Q is placed. Thus, immediately after ordering, the inventory position is between R and $R+Q$. Let $n = \lceil \frac{R-R_e}{Q} \rceil$ express the smallest integer greater than $\frac{R-R_e}{Q}$. Hence, the largest amount of a single replenishment order is nQ if all n orders are placed with a single supplier. Incorporating the concept of delivery splitting, we allow for each replenishment order of size i^*Q (i = 0,1,2, ……, n-1) to be delivered in i batches of size Q. This inventory control policy can also be described in terms of the inventory level (stock-on-hand). Hence, an alternative description for the control policy in terms of the inventory level is as follows: the policy requires placing orders of size Q for every down crossing of level R-iQ (i=0,1,2, ……,n-1) when tracing the inventory level process. This implies that when the inventory level is between $\max(R_e+1, R-iQ)$ and $(R+Q-iQ)$, there are i $(i=0,1,,\ldots,n)$ outstanding orders, each with the size Q. The replenishment lead time for regular order is exponentially distributed with a mean of σ^{-1}. The replenishment lead times of orders are independent.

An emergency order is placed when the inventory level becomes less than or equal to the emergency reorder point R_e. The corresponding order quantity is uQ_e is the smallest positive integer that can push the inventory level above R_e. Therefore,$u=\left\lfloor \frac{(k-w+R_e)}{Q_e}+1 \right\rfloor$, where w is the inventory level just before a surge demand of size k units. Note that, if the inventory level w, at the time when surge demand arrives, is above R and if the size of surge demand is $k \geq w-R_e$, then both regular orders and an emergency order are placed. The lead time for the emergency orders is assumed to be zero. The model assumes that there is a shortage cost when the inventory level goes below R_e+1.

3.1. Modeling framework

The primary goals of this subsection are to develop the stationary distribution function of each inventory level and develop the total cost function. LCT is used to derive the stationary distribution function of inventory level w. In order to apply LCT, it is necessary to define the State, Sample Path, Level, Up Crossing and Down Crossing of the system. These definitions of State, Sample Path, Level, Up Crossing and Down Crossing are similar to Roni et al. (2015). The LCT starts with constructing a sample path of the process over time. An important step of the LCT is to construct a typical sample path of the underlying stochastic process. Correct construction of a sample path requires a thorough understanding of the dynamics of the model. Although in many applications the construction of sample paths is straightforward, the construction of sample paths may be a nontrivial or even a challenging task in complex models with state dependencies . Let $\{W(t), t>0\}$ be the inventory level at time t and $D(w,t)$ be the duration of the inventory level at w up to time t. The inventory at any instant can be between $w \in [R_e+1, R+Q]$. Let $P(w)=\lim_{t\to\infty}\frac{D(w,t)}{t}$ denote the asymptotic probability of inventory level w. When $W(t) \in [\max(R_e+1, R-iQ), R+(i-1)Q]$ there are i $(i=0,1,\ldots,n)$ outstanding orders. A sample path is a typical tracing of the inventory level, $W(t)$, over time. A typical sample path is shown with numerical values in Fig. 1. The sample path shows how the inventory level changes over time. The sample path for $W(t)$ goes down by small jumps, and sometimes it goes down by big jumps (as shown in Fig. 1). The sample path is generated for the case when $R=20, Q=5, R_e=5,$ and $Q_e=3$. The inventory level starts with $R+Q=25$. When either the regular demand or surge demand arrives, inventory level goes down. Regular and surge demand arrival can be visualized by the decrease of inventory level. These decreases in the inventory level by a small jump or by a big jump are caused by the arrival of regular and surge demand respectively.

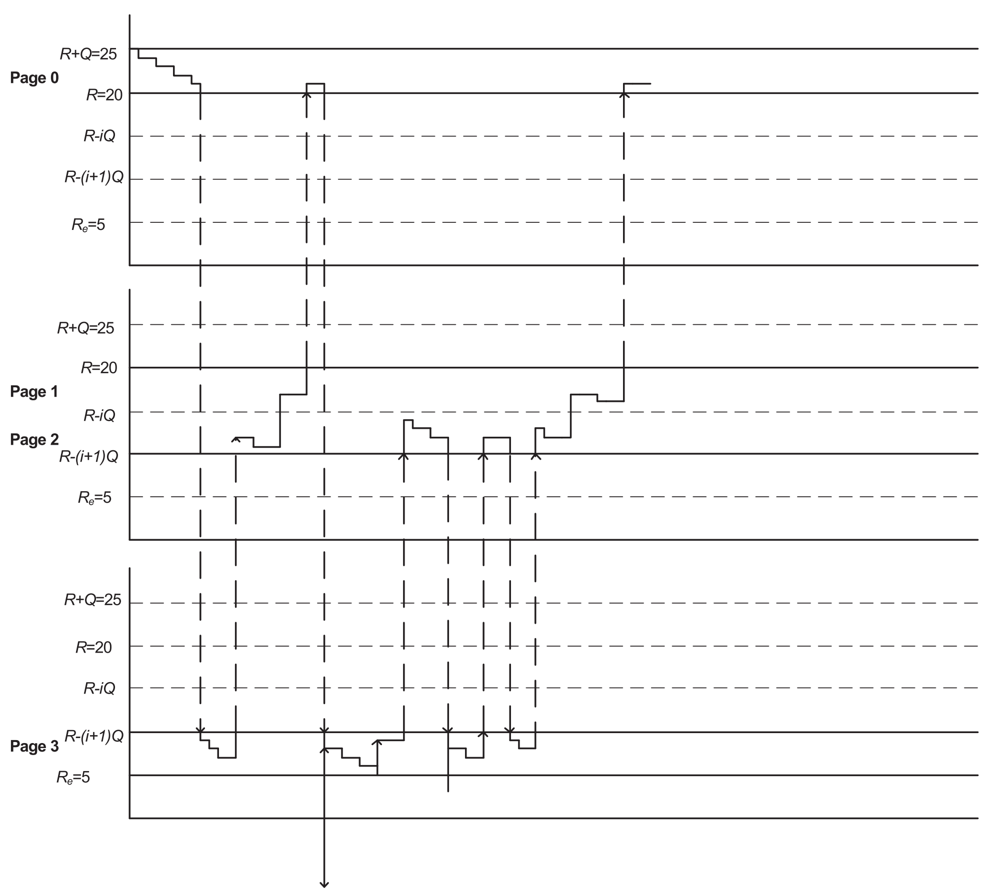

Fig. 1. Sample of path of typical inventory level.

The sample path goes up when regular and emergency orders are received. The state space of S can be divided into sub spaces based on the number of outstanding regular orders. These sub spaces are called *Pages*. Initially, when the inventory level $W(t)\in[R+1,R+Q]$, there is no outstanding order. This sub space is called page-0. When inventory level $W(t)\in[R-Q+1,R+1]$, there is one outstanding order. So, the subspace $w\in[R-Q+1,R+1]$ is called page-1. Similarly, when $(t)\in[\max(R_e+1,R-iQ),\ R+(i-1)Q]$, there are i $(i=0,1,\ldots,n)$ pages. Fig. 1 shows 4 pages and their corresponding inventory level ranges.

3.2. Development of the balancing equations

Now we will develop the balancing equation of each page $i=0,1,\ldots,n$.

3.2.1. Page-0

In page-0, the sample path enters into the following levels $w\in[R+1,R+Q]$. Page-0 has two types of down crossing, unit down crossings caused by regular unit demand and jump down crossings caused by surge demand. In this page, up crossings only occur due to the receipt of regular orders. Let α define the size of a demand that causes down crossing of inventory level w or the amount of received order that causes up crossing of inventory level w. Based on LCT, the balance equation for page-0 can be formulated by Eq. (1).

$$\lambda_1 P(w)+\lambda_2\sum_{\alpha=w}^{R+Q}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha) = \sigma\sum_{\alpha=(w-Q)}^{R}P(\alpha)\,\forall w\in[R+1,R+Q] \tag{1}$$

In Eq. (1), $\lambda_1 P(w)$ is the down crossing rate due to regular demand, $\lambda_2\sum_{\alpha=w}^{R+Q}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha)$ is the down crossing rate due to surge demand. At the right hand side $\sigma\sum_{\alpha=(w-Q)}^{R}P(\alpha)$ is the up crossing rate due to regular order replenishment.

3.2.2. Page-$i=1,2,\cdots,n-1$

For each page $i=1,2,\cdots,n-1$, the sample path is in the following levels $w\in[R+Q-(i+1)Q+1,R+Q-iQ]$. This page also has two types of down crossings, unit down crossings and jump down crossings. In page-i jump down crossings could also be caused by a surge demand in any page-j, where $j=0,1,\cdots,i-1$. In page-i up crossings occur due to both regular order replenishments from page-i and page-$(i+1)$. The balance equation is formulated by Eq. (2).

$$\lambda_1 P(w)+\lambda_2\sum_{\alpha=w}^{R+Q-iQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha) +\lambda_2\sum_{j=0}^{i-1}\sum_{\alpha=R+Q+1-(j+1)Q}^{R+Q-jQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha) \tag{2}$$

$$=i\sigma\sum_{\alpha=R+Q-(i+1)Q+1,}^{w-1}P(\alpha)+(i+1)\sigma\sum_{\alpha=\max(w-Q,\,R_e+1)}^{R+Q-(i+1)Q}P(\alpha)$$

$$\forall w\in[R+Q-(i+1)Q+1,R+Q-iQ],\ i=1,2,\cdots,n-1$$

On the left hand side of Eq. (2), $\lambda_1 P(w)$ is the down crossing rate due to regular demand on page-i, $\lambda_2\sum_{\alpha=w}^{R+Q-iQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha)$ is the jump down crossing rate into interval $[w,R+Q-iQ]$ from page-i , and $\lambda_2\sum_{j=0}^{i-1}\sum_{\alpha=R+Q+1-(j+1)Q}^{R+Q-jQ}\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k P(\alpha)$ is the jump down crossing rate in the interval $[R+Q-(i+1)Q+1,R+Q-iQ]$ from page $j=0,1,\cdots,i-1$. The first term of the right hand side, $i\sigma\sum_{\alpha=R+Q-(i+1)Q+1,}^{w-1}P(\alpha)$ is the up crossing rate due to regular order replenishment in page-i. The second term, $(i+1)\sigma\sum_{\alpha=\max(w-Q,R_e+1)}^{R+Q-(i+1)Q}P(\alpha)$ on the right hand side of Eq. (2) expresses up crossing rate due to regular order replenishment from page-$(i+1)$.

3.2.3. Page-n

In page-n, the sample path enters into the range of $w\in[R_e+1,R+Q-nQ]$. Similar to page-i $(i=1,2,\cdots n-1)$, page-n has unit down crossings and jump down crossings due to regular and surge demands. Unlike page-0 through page-$(n-1)$, depending on the inventory level, up crossings in page-n may also be caused by emergency order replenishment. For the inventory level $w\in[R_e+1,R_e+Q_e]$ up crossings are caused by both regular order replenishment and emergency order replenishment. For $w\in[R_e+Q_e+1,R+Q-nQ]$ up crossings are only caused by regular orders. Thus, balance equations for page-n in the interval $w\in[R_e+1,R_e+Q_e]$ and $w\in[R_e+Q_e+1,R+Q-nQ]$ can be formulated by (3a) and (3b).

$$\lambda_1 P(w)+\lambda_2\sum_{\alpha=w}^{R+Q-nQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha) +\lambda_2\sum_{j=0}^{n-1}\sum_{\alpha=R+Q+1-(j+1)Q}^{R+Q-jQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha)$$
$$=n\sigma\sum_{\alpha=R_e+1}^{w-1}P(\alpha)+\lambda_2\sum_{\alpha=R_e+1}^{R+Q}\sum_{m=1}^{int\left\{\frac{b-\alpha+R_e}{Q_e}\right\}+1}\sum_{k=\max\{a,(m-1)Q_e+\alpha-R_e\}}^{k=\min(b,mQ_e+\alpha-w)} r_k\,P(\alpha)+\lambda_1 P_1(R_e+1)$$
$$\forall w\in[R_e+1\ \ R_e+Q_e] \tag{3a}$$

$$\lambda_1 P(w)+\lambda_2\sum_{\alpha=w}^{R+Q-nQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha) +\lambda_2\sum_{j=0}^{n-1}\sum_{\alpha=R+Q+1-(j+1)Q}^{R+Q-jQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha)$$
$$=n\sigma\sum_{\alpha=R_e+1}^{w-1}P(\alpha)\quad\forall w\in[R_e+Q_e+1,R+Q-nQ] \tag{3b}$$

On the left hand side of Eqs. (3a) and (3b), $\lambda_1 P(w)$ denotes the down crossing rate due to regular demand on page n, $\lambda_2\sum_{\alpha=w}^{R+Q-nQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha)$ is the jump down crossing rate into interval $w\in[w,R+Q-nQ]$ from page n, and $\lambda_2\sum_{j=0}^{n-1}\sum_{\alpha=R+Q+1-(j+1)Q}^{R+Q-jQ}\left(\sum_{k=\max\{\alpha-w+1,a\}}^{b} r_k\right)P(\alpha)$ is the jump down crossing in $w\in[R+Q-(i+1)Q+1,R+Q-iQ]$ from page j = 0, 1, 2, n-1. If we look at the right hand side of (3a) and (3b), the term $\sigma\sum_{\alpha=R_e+1}^{w-1}P(\alpha)$is the up crossing rate due to regular order replenishment. The term $\lambda_2\sum_{\alpha=R_e+1}^{R+Q}\left(\sum_{n=1}^{int\left\{\frac{b-\alpha+R_e}{Q_e}\right\}+1}\sum_{k=\max(a,(n-1)Q_e+\alpha-R_e)}^{\min(b,nQ_e+\alpha-w)} r_k\right)P(\alpha)$ in (3a) expresses up crossing rate due to emergency order replenishment. At $w=(R_e+1)$, if there is a unit regular demand, it will trigger an emergency order. The term $\lambda_1 P(R_e+1)$ on the right hand side of Eq. (3a) expresses the up crossing rate due to emergency orders triggered by regular demand arrival at $w=(R_e+1)$. Since this emergency order replenishment will raise the inventory level to R_e+Q_e from R_e, the up crossing rate $\lambda_1 P_1(R_e+1)$ is only applicable for $w \in [R_e+1, R_e+Q_e]$.

By adding normalization Eq. (4) we can solve the linear systems of Eqs. (1)–(4).

$$\sum_{w=R_e+1}^{R+Q} P(w) = 1 \tag{4}$$

3.3. Formulation of the total cost function

The stationary distribution of the inventory level can be easily found by solving Eqs. (1)–(4), which are a set of linear equations. The steady state inventory level can be used to calculate long-run average costs. Let $\mathrm{TC}(R, Q, R_e, Q_e)$ denote the long-run average of the total cost per time unit, which consists of regular order cost, emergency order cost, inventory holding cost, and shortage cost per unit time under a given policy (R, Q, R_e, Q_e). Due to PASTA (Wolff, 1982) and the memory-less property of the exponential distribution, we can write long-run average total cost per time unit in (5).

$$\mathrm{TC}(R, Q, R_e) = hE(IL) + \lambda_1 K_1 P_{RO} + \lambda_2 K_1 P_{RO} + \lambda_1 K_2 P_{EO} + \lambda_2 K_2 P_{EO} + \lambda_2 SE(S) \tag{5}$$

Here, h is the unit inventory holding cost per time unit, $E(IL)$ is the expected on-hand inventory level, K_1 is the unit regular order cost,P_{RO} is the probability of regular order, K_2 is the fixed emergency cost, P_{EO} is the probability of emergency order, S is the shortage cost per unit and $E(S)$ is the expected shortage. Eqs. (6)–(11) provide details for calculating each cost term in (5)

$$hE(IL) = h\sum_{w=R_e+1}^{R+Q} wP(w) \tag{6}$$

$$\lambda_1 K_1 P_{RO} = K_1\left\{\lambda_1 \sum_{i=0}^{n-1} P(R-iQ+1)\right\} \tag{7}$$

$$\lambda_2 K_1 P_{RO} = \lambda_2 K_1 \sum_{j=0}^{n} \sum_{w=R-jQ+1}^{R+(1-j)Q} \sum_{k=\max(a, w-(R-jQ))}^{b} r_k P(w) \tag{8}$$

$$\lambda_1 K_2 P_{EO} = \lambda_1 K_2 P(R_e+1) \tag{9}$$

$$\lambda_2 K_2 P_{EO} = \lambda_2 K_2 \sum_{w=R_e+1}^{R+Q} \left(\sum_{k=\max\{w-R_e, a\}}^{b} r_k\right) P(w) \tag{10}$$

$$\lambda_2 SE(S) = \lambda_2 S \sum_{w=R_e+1}^{R+Q} \left(\sum_{k=w+1}^{b} (k-w) r_k\right) P(w) \tag{11}$$

Based on the above total cost function (Eqs. (5)–(11)), we developed a search based heuristics in the next section to find near optimal values of R, Q, R_e if Q_e is given.

4. Optimization using a search-based heuristics

We assume that Q_e, is given. In practice, Q_e is often the minimum measurement unit and is fixed. For a given policy (R, Q, R_e), we can easily calculate the total cost using Eqs. (1)–(11). The balancing equations developed by Roni et al. (2015) were based on two pages. There were two pages in the model presented in Roni et al. (2015) because the maximum number of outstanding regular orders was one. However, in this paper we allow split delivery. Incorporating the concept of delivery splitting, we allow for each replenishment order of size i^*Q (i=0,1,2, ……, n-1) to be delivered in i batches of size Q. When we split the delivery, the maximum number of outstanding regular orders would be i=0,1,2, ……,n-1. For each page, we develop balancing equations as shown in Eq. (2). This increases the number of balancing equations set in this paper. The total cost function in the model with balancing equations (Eqs. (1)–(4)) are not linear programming with embedded nonlinear functions of int(), max(), and min(). The linearization requires a number of additional variables and constraints with respect to Roni et al. (2015). The additional variables are mostly binary. Solving such a model using a commercial solver such as CPLEX would increase the computational time substantially. Therefore, in order to find a near-optimal inventory policy for this model we propose a heuristics based on Tabu search. Tabu search explores the region of feasible solutions using a descent strategy guided by several control rules. A detailed description of the basic Tabu search heuristic can be found in Glover (1989, 1995). The main components of the Tabu search algorithm are the neighborhood definition, the search memories or rules (size of Tabu list, and aspiration criteria), the diversification procedure, and the stopping criterion. The neighborhood defines the set of potential movements to choose from in each iteration.

4.1. Neighborhood definition

The n-step neighborhood of a feasible solution is defined as the set of other feasible solutions which can be reached in n steps. Let $x_1, x_2, \ldots\ldots x_m$ denote a set of feasible policies for a model of m decision variables and let n be an integer number. The following is a formal formulation of the n-step neighborhood provided for a feasible solution of size m: $N(x_1, x_2, \ldots\ldots x_m) = \{x_1, \ldots\ldots x_m) | x_i \in S_i,\ i = 1, 2, 3, \ldots.., m\}$ where $S_i = \{x_i \pm k | k = 1, 2, \ldots.n\}$. For our problem, we first determine the set of one-step neighborhood of a feasible inventory policy. The one-step neighborhood for a policy $R^0, Q^0$and $R_e{}^0$ is given by

$$\begin{aligned}
N(R, Q, R_e) = \{&(R+1, Q+1, R_e+1), (R+1, Q+1, R_e),\\
&\times(R+1, Q+1, R_e-1), (R, Q, R_e-1), (R+1, Q, R_e+1),\\
&\times(R+1, Q, R_e), (R+1, Q, R_e-1), (R+1, Q-1, R_e+1),\\
&\times(R+1, Q-1, R_e), (R+1, Q-1, R_e-1),\\
&\times(R+1, Q-1, R_e+1), (R+1, Q-1, R_e),\\
&\times(R+1, Q-1, R_e-1), (R, Q+1, R_e+1),\\
&\times(R, Q+1, R_e), (R, Q+1, R_e-1),\\
&\times(R-1, Q+1, R_e+1), (R-1, Q+1, R_e),\\
&\times(R-1, Q+1, R_e-1), (R-1, Q-1, R_e-1),\\
&\times(R-1, Q-1, R_e+1), (R-1, Q-1, R_e),\\
&\times(R, Q-1, R_e-1), (R-1, Q, R_e-1),\\
&\times(R, Q, R_e+1)(R, Q-1, R_e)\}
\end{aligned}$$

We eliminate infeasible inventory policies from the neighborhood by inspecting the value of R, Q, R_e. We know that $> R_e$. Therefore, we pick a neighborhood $N(R, Q, R_e)$ in a such way that $R > R_e$.

4.2. Tabu search procedure

Tabu search is guided by a Tabu list, which is used to avoid visiting some inventory policies that have already been examined. The list keeps a record of the most recent policies visited, as well as, the best policies found. The policies of the Tabu list are sorted based on their total cost in order to facilitate add, delete and update operations. The Tabu list has a maximum size. When the limit is reached, a new solution is added to the list and one solution is removed.

The algorithm starts by building an initial solution P_0. The one-step neighbors of this solution $P_c \in N(P_0)$ are identified and evaluated. If a neighbor is not in the Tabu list, then it is added to this list. If the list is already full, then the policy with the largest total cost (located in the bottom of the list) is removed. If the neighboring solutions are not better than P_0, then Tabu search explores other options to improve the quality of solutions explored. This is achieved by using a diversification procedure. This procedure selects the worst

Step 1: Set $iter = 1$; Initial feasible policy = P_0; Tabu list= $\{P_0\}$; Compute $TC(P_0)$;
$best\_cost = TC(P_0)$,
Step 2: a) $improved$ = no;
b) Find all feasible policies P_c from $N(P_0)$.
c) For each policy $P_c \in N(P_0)$
If P_c is not in the Tabu list,
Then add or update the Tabu list
If $TC(P_c) < best\_cost$
Then $P_0 = P_c$, $best\_cost = TC(P_0)$, $improved$ = yes
Else Go To next P_c
Step 3: If $improved$ = yes, then go to **Step 2**, Else go to **Step 4**.
Step 4: a) Find the policy P_w with the highest TC from the Tabu list; $local\_best = TC(P_w)$
b) Remove P_w from the Tabu list;
c) Find all feasible policy P_c from $N(P_w)$
d) For each policy $P_c \in N(P_w)$
If P_c is not in the Tabu list
Then add or update the Tabu list;
If $TC(P_c) < local\_best$ Then $P_l = P_c$; $local\_best = TC(P_l)$
Else Go To next P_c ;
Step 5: a) If $local\_best < best\_cost$, then $best\_cost = local\_best$ Else $iter = iter + 1$;
b) $P_0 = P_l$;
Step 6: Stop if the stopping criteria are satisfied; otherwise go to **Step 2.**

Fig. 2. The Tabu-search procedure.

Initialize:
$Q^u = Q, Q^l = 1, R_e = 0, n = 1, \Delta = 1$
While $(Q^u - Q^l < 1)$ do
Begin
$Q = \left\lceil \frac{Q^u + Q^l}{2} \right\rceil$
Compute $M(R) = min\{TC(R, Q, R_e) | R_e < R < nQ\}$
Compute $M(R, \Delta) = min\{TC(R, Q - \Delta, R_e) | R_e < R < nQ\}$
If $(M(R, \Delta) > M(R))$
$Q^l = Q$;
$n = \left\lceil \frac{R}{Q^l} \right\rceil$;
Else
$Q^u = Q$
$n = \left\lceil \frac{R}{Q^u} \right\rceil$;
End
End
Return R, Q, R_e

Fig. 3. The procedure for finding an initial feasible policy.

policy P_w from the Tabu list. The one-step neighborhood of P_w is then explored. The algorithm stops after a maximum number of iterations without improvements in the total cost. A step-by-step description of the heuristic is presented in Fig. 2.

The initial solution which starts the algorithm is not selected at random. Instead, a binary search procedure is developed in order to obtain this solution. The procedure is designed in such a way that it identifies a good quality initial feasible solution when $R_e = 0$. Let Q^u and Q^l denote the upper bound and lower bound of optimal order quantity. The proposed procedure divides the range $\left[Q^l, Q^u\right]$ into equal parts (for example, $\frac{Q^l + Q^u}{2}$) and identifies the segment where the local optimal order quantity Q lies. The value of $TC(R, Q, R_e)$ is used to identify the segment which contains the local optimal order quantity Q. Let $M(R)$ denote the minimum value of $TC(R, Q, R_e)$ for given Q,R_e where $R_e < R < nQ$. Let Δ denote the amount by which we modify the value of Q, and $M(R, \Delta) = TC(R, Q - \Delta, R_e)$. If $(M(R, \Delta) > M(R))$ then $Q = Q^l$. This procedure is repeated until the width of range $\left[Q^l, Q^u\right]$ decreases to 1. Fig. 3 presents details of the procedure that is used to find the initial feasible policy.

5. Numerical experiment and results

In order to demonstrate the applicability of the proposed heuristics, we conducted a numerical study. There are no benchmark data sets in the literature for this particular problem that we could directly use to test and compare the robustness of the algorithm. To demonstrate the efficiency of the algorithm in terms of solution time and quality, we solve our model using randomly generated data sets. The distributions are used to demonstrate regular and surge demand pattern. In order to evaluate the performance of the proposed heuristic, we compare the solutions generated from the heuristic with the corresponding optimal solutions found from explicit enumeration.

5.1. Experimental design

The Tabu search procedure is implemented with C++ Visual Studio 2010 on a Duo 2 Core 2.67 GHz PC with 4 GB of RAM. 10 different scenarios are generated and used to test the performance of the heuristic. Each scenario consists of a set of model input parameters. The parameters used for each scenario are listed in Table 1. The probability distribution of r_k is defined by (12). The probability distribution of r_k is such that r_k decreases as the value of k increases.

$$r_k = \frac{2(b-k)}{(b-a)(b-a+1)} \text{ where } k \in [a, b] \tag{12}$$

The probability distribution function in Eq. (12) captures the size and nature of surge demand. In reality, small incidents, such as car accidents are more likely to bring low demand volumes than big incidents such as nuclear disasters. As indicated from Eq. (12), the value of r_k decreases with the increase in k. However, any other discrete probability distribution could be used to model the surge demand r_k when the method discussed in Section 3 is applied. The numerical experiments find near optimal values for R, Q, R_e.

We examine all possible one-step neighbors of a policy. Numerical experiments are designed to analyze the performance of the heuristics for different values of problem parameters. The numerical experiments are designed in order to identify the impact of the maximum number of iterations, the size of Tabu list, and the neighborhood structure on the quality of solutions found.

Table 1
Problem parameters.

Scenario	λ_1	λ_2	σ	h	K_1	K_2	a	b	Q_e	s
1	1	0.02	1.5	0.8	25	140	2	30	3	100
2	3	0.02	1.5	0.8	25	140	2	40	3	200
3	1	0.02	1.5	1	25	140	2	40	3	200
4	2	0.02	1.5	1	25	140	2	40	3	200
5	2	0.02	1.5	1	25	140	2	50	3	200
6	5	0.1	4	1	25	140	2	50	3	250
7	7.5	0.4	7	1	40	160	2	50	3	500
8	10	0.8	9	1.2	40	160	2	80	3	1000
9	12	1	10	1.2	40	160	2	100	3	1200
10	15	1	12	1.2	40	160	2	100	3	1500

Table 2
Summary of the results from the Tabu search heuristics.

Scenario	R^*	Q^*	R_e^*	Near Opt. cost found from Tabu search heuristics ($)	Relative difference with optimal solutions found from total enumeration (%)
1	12	15	0	20.77	0.09
2	16	20	0	29.14	0.96
3	16	14	1	30.48	1.03
4	14	15	0	32.25	1.40
5	21	16	2	38.52	1.68
6	33	24	0	60.40	4.9
7	46	44	0	86.03	5.8
8	94	45	17	178.56	6.1
9	122	59	45	229.32	6.4
10	126	107	87	273.39	7.2

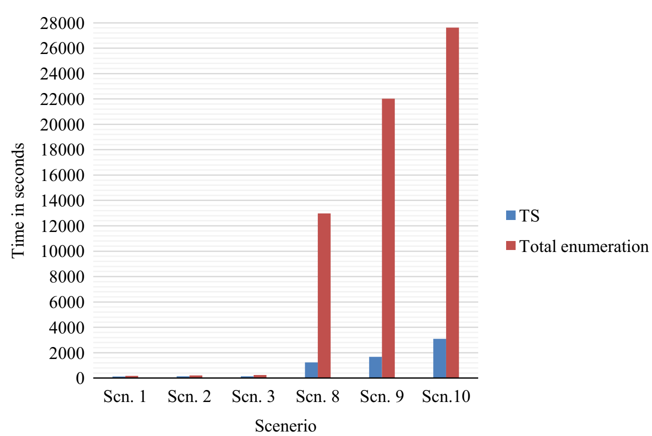

Fig. 4. Running time of Tabu search and total enumeration.

5.2. Performance of the Tabu search heuristics

Table 2 provides a summary of the results from the Tabu search heuristics. The performance of the procedure is evaluated using two criteria: solution quality and CPU running time. In order to evaluate the quality of the solutions, we compare them to the optimal cost found using the total enumeration technique. The procedure for total enumeration is shown in Appendix A. The relative difference ($\Delta\%$) between the two solutions is measured using this formula

$$\Delta\% = \frac{NearoptimalSol.usingtabusearch - OptimalSol.bytotalenumeration}{OptimalSol.bytotalenumerationl} \times 100\%.$$

Our computational results indicate that the Tabu search procedure provides high quality solutions. Our numerical analysis shows that depending on the scenario, the relative difference between the total enumeration and the Tabu search heuristics could be 0.09–7.2% as shown in Table 2. The running time of Tabu search heuristics is smaller

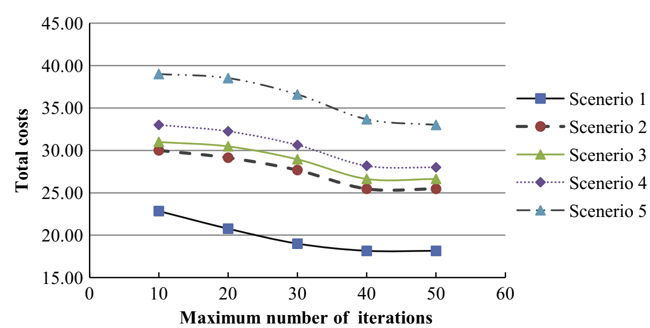

Fig. 5. Maximum number of iterations versus total costs.

compared to total enumeration method. As indicated in Fig. 4, the running time of explicit enumeration is higher than the Tabu search algorithm, especially when the problem size increases. This implies that simple total enumeration is reasonable for small instance but a search heuristics is necessary for large scale instances.

5.3. Heuristics-related parameter selection

The maximum number of iterations without an improvement is used as a stopping criterion of the Tabu search heuristics. We ran the experiments using different values for the maximum number of iterations. Fig. 5 presents the relationship between the total system cost and the maximum number of iterations for each scenario. The figure shows that the maximum number of iterations has some effect on the performance of the proposed heuristics. However, increasing the maximum number of iterations has a great impact on the running time of the algorithm. We also recorded the iteration number at which the best solution was obtained. Fig. 6 presents the iteration number where the best solution was obtained under different maximum number of iterations and scenarios. The results indicate that most of the final solutions are obtained within 30 iterations.

It is well-known that the size of the Tabu list affects the performance of the heuristics (Glover, 1995). If the size of Tabu list is small, then, the search may be confined to a local optimum. Small Tabu lists prevent the search process from jumping around when exploring the solution space. In contrast, if the size of the Tabu list is too big, then additional computational time is required to add, delete and update the list. Thus, the search procedure spends a good portion of the computational time updating the Tabu list instead of exploring the solution space. In order to evaluate the impact of the size of Tabu list we ran the experiments and identified the best policy for a fixed list size. We used five different sizes which are: 10, 20, 30, 40 and 50 respectively. Fig. 7 summarizes the results of the experiments. One can see that, as the size of the Tabu list grows, the policy costs decrease. However, the decrease in costs is very small when the size of the Tabu list is larger than 30.

5.4. Performance of the hybrid inventory policy with split delivery

Roni et al. (2015) proposed a hybrid policy for a stochastic inventory system facing regular and surge demand with standard delivery. They investigated the benefit of using the hybrid inventory model over the basic inventory policy without emergency orders. They showed that the emergency reorder point R_e is sensitive to unit shortage cost s and increases as unit shortage cost s goes up. We can see the similar result in our hybrid inventory policy with split delivery. The emergency reorder point R_e is high in the scenarios where unit shortage cost s is high as shown in scenarios 8, 9, 10 (Table 2). However, the behavior of the proposed hybrid inventory policy with split delivery is different than a hybrid inventory policy with standard delivery proposed by Roni et al. (2015). Our numerical analysis indicates that considerable cost savings are obtained from split delivery depending on various instances. Table 3 shows that the hybrid inventory model with

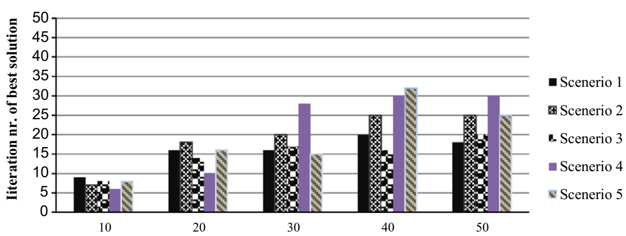

Fig. 6. Maximum number of iterations versus the number of iterations to find the best solution.

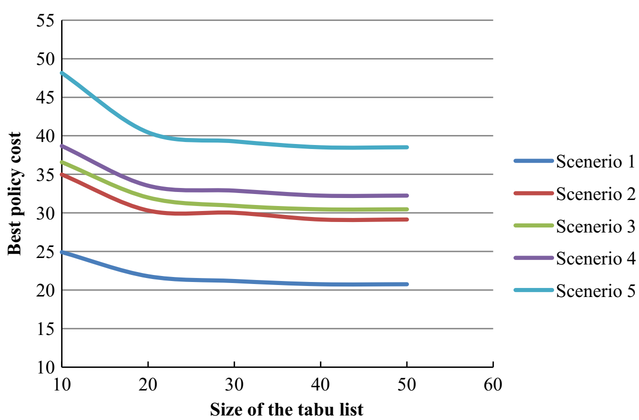

Fig. 7. Size of Tabu list versus total cost.

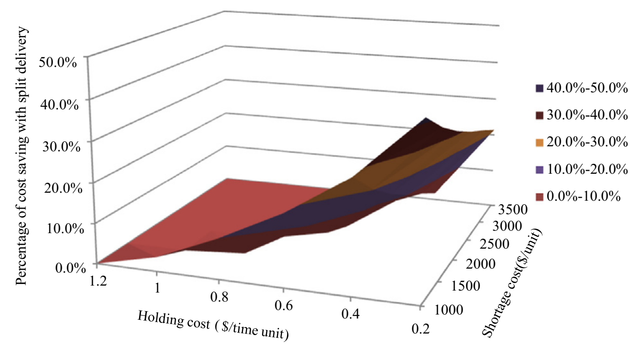

Fig. 8. Percentage of cost saving of the hybrid policy with split delivery, compared to the standard delivery, as a function of holding and shortage costs for scenario 8.

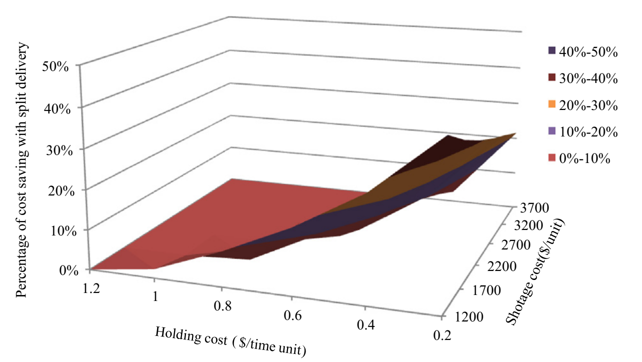

Fig. 9. Percentage of cost saving of the hybrid policy with split delivery, compared to the standard delivery, as a function of holding and shortage costs for scenario 9.

Table 3
Comparing the performance of the hybrid inventory policy with split delivery with the standard delivery policy.

Scenario	λ_1	λ_2	σ	K_1	K_2	a	b	Q_e	s	h	Near optimal cost with standard delivery ($/time unit)	Near optimal cost with split delivery ($/time unit)	Percentage of cost savings (%)
8	10	0.8	9	40	160	2	80	3	1000	1	155.50	149.99	3.5
									1500	0.4	130.11	103.48	20.5
									2000	0.4	153.96	131.08	14.9
									2500	0.4	175.13	157.29	10.2
									3000	0.2	145.79	109.48	24.9
9	12	1	10	40	160	2	100	3	1000	1	199.39	194.92	2.2
									1500	0.8	223.03	220.91	0.9
									2000	0.2	151.49	103.93	31.4
									2500	0.2	170.42	122.83	27.9
									3000	0.2	187.78	141.03	24.9
10	15	1	12	40	160	2	100	3	1500	0.8	255.98	250.20	2.3
									2000	0.6	258.42	243.94	5.6
									2500	0.4	243.82	212.23	13.0
									2500	0.2	188.96	132.64	29.8
									3000	0.2	206.28	150.33	27.1

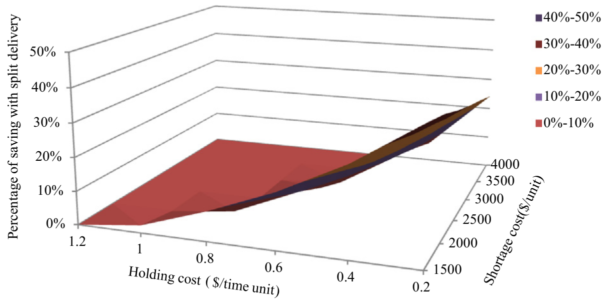

Fig. 10. Percentage of cost saving of the hybrid policy with split delivery, compared to the standard delivery, as a function of holding and shortage costs for scenario 9.

split delivery could save up to 31.4% of costs across instances compared to standard delivery. Note that that the scenario analysis is only a heuristic method we use in order to estimate performance of the hybrid inventory policy with split delivery.

Numerical results show (Figs. 8, 9 and 10) that hybrid inventory policy with split delivery outperforms hybrid inventory policy with standard delivery for relatively low shortage and holding costs. But hybrid inventory policies with split delivery tends to be expensive when both the shortage and holding cost are high. Figs. 8–10 show the percentage of cost saving in hybrid policy with split delivery with respect to standard delivery as a function of holding and shortage cost for scenarios 8–10. For example, Fig. 8 shows that as the shortage cost goes down from \$3500 to \$1000 per unit and holding cost goes down from \$1.2 to \$0.2 per time unit, the hybrid inventory model with split delivery could save up to 42.1% with respect to standard delivery in scenario 8. We can see similar cost saving in scenario 9 and 10 respectively as shown in Figs. 9 and 10.

6. Conclusion and future research

This paper proposes a hybrid inventory policy with split delivery under regular and surge demand. Split delivery considers the scenario where there is a single supplier who replenishes all orders but the supplier could split the delivery of the orders using multiple shipments. This paper identifies an inventory management policy that minimizes costs while maintaining a high service level. The level crossing theory is used to formulate this problem and calculate the equilibrium equations for the stationary inventory level probability density function (PDF) under a given inventory policy featured by the ordering points and order quantities of both order types. A Tabu search-based heuristics is developed in order to identify solutions for this problem. Numerical experiments demonstrate that the Tabu search finds high quality solutions in a shorter time than the explicit enumeration. Numerical results generated from different instances also show that proposed hybrid inventory policy with split delivery could outperform hybrid inventory policies with standard delivery by saving 3.5–42.1% of total cost.

The paper only considers a single-item single-location inventory system. One future research direction could be to extend the model to a multiple-location case. The presented model assumes that items are not perishable, though many medicines have finite shelf life in practice. This hybrid inventory model could be extended to perishable products with a fixed or variable shelf life. The optimization approach relies on Tabu search based heuristics. Therefore, another potential extension is developing exact solution algorithms which find optimal inventory policies. In summary, this paper makes a contribution to the inventory management literature regarding the response to two types of demand patterns. In practice, the model and solution approach presented in this paper can help healthcare providers or humanitarian logistics providers in managing their inventories. The proposed hybrid inventory policy with split delivery could further reduce inventory cost in a system that faces regular and surge demands.

Appendix A

Procedure: Total enumeration

Initialize
Optimal cost $= \infty$
$R^* = 0$
$Q^* = 0$
$R_e{}^* = 0$

For $R = 1$ to R_{max} **Do**
  For $Q = 1$ to Q_{max} **Do**
    For $R_e = 0$ to R_{max} **Do**
      If($\text{TC}(R, Q, R_e) <$ Optimal cost **Then**
        Optimal cost $= \text{TC}(R, Q, R_e)$
        $R^* = R$
        $Q^* = Q$
        $R_e{}^* = R_e$
    Next R_e
  Next Q
Next R

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