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A hybrid inventory management system responding to regular demand and surge demand ☆

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ABSTRACT

This paper proposes a hybrid policy for a stochastic inventory system facing regular demand and surge demand. The combination of two different demand patterns can be observed in many areas, such as healthcare inventory and humanitarian supply chain management. The surge demand has a lower arrival rate but higher demand volume per arrival. The solution approach proposed in this paper incorporates the level crossing method and mixed integer programming technique to optimize the hybrid inventory policy with both regular orders and emergency orders. The level crossing method is applied to obtain the equilibrium distributions of inventory levels under a given policy. The model is further transformed into a mixed integer program to identify an optimal hybrid policy. A sensitivity analysis is conducted to investigate the impact of parameters on the optimal inventory policy and minimum cost. Numerical results clearly show the benefit of using the proposed hybrid inventory model. The model and solution approach could help healthcare providers or humanitarian logistics providers in managing their emergency supplies in responding to surge demands.

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1. Introduction

Natural disasters (e.g., earthquakes, disease outbreak, volcanic eruptions, floods, tsunamis, storms, and hurricanes) and man-made events (e.g., massive accidents and terrorist attacks) may create surge demand for medicines or emergency items. Though the frequency of surge demand is very low, the demand volume per event could be high. Providing humanitarian relief supplies, such as water, food, medicines and associated services, to victims could be a major challenge due to the unpredicted nature of surge demand for these items and services. For example, in 2011 Haiti experienced an earthquake that was the biggest earthquake in the last two centuries and affected the capital city of Port-au-Prince. This earthquake frustrated the country because of the lack of a well-planned and coordinated distribution of relief supplies for weeks [1]. For this reason, we believe that disaster management teams are in need of tools that can help with responding in a timely manner to disasters by providing the necessary supplies and services [2]. A well-managed disaster planning could save lives, reduce financial loss, protect assets, facilitate recovery, and provide greater security and safety.

Since many medical items are critical to saving people's lives, healthcare providers or humanitarian organizations need to keep enough inventories for those incidents [3]. Managing the inventory of healthcare or humanitarian service related products is more challenging as compared to managing the inventory of products for regular manufacturing. In traditional manufacturing, if the demand forecasts are inaccurate or the order replenishment does not follow the plan, the results are delayed orders or at worst canceled orders. However, in healthcare delivery or humanitarian service, a shortage could mean a difference between life and death. Therefore, healthcare providers often have to maintain high service levels for life-saving items. This is achieved either by increasing the inventory level or through expedite shipments. Both approaches are expensive. In order to minimize the inventory holding costs, while maintaining high service levels, healthcare providers and their suppliers need to develop sophisticated and proactive inventory management policies for items that may face surge demand. When surge demand occurs in one region, rapid delivery of emergency items to the affected area is important. Such a short-term immediate response could incur high costs. A proactive inventory management system could minimize the occurrence of these shortage costs. Motivated by the issues stated above, this paper proposes a continuous-review hybrid inventory policy that relies on using regular orders and emergency orders in response to regular

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and surge demands to support humanitarian operations. The paper uses the level crossing theory (LCT) to obtain the long-run average cost under a given policy. A mixed integer program is developed to optimize the inventory policy parameters, including regular order point, emergency order point, and regular order quantity.

This paper contributes to the literature in the following ways. First, it provides a hybrid stochastic inventory model responding to regular demand and surge demand. An analytical model based on LCT is developed to derive the stationary distribution of the inventory level for a continuous-review inventory system characterized by 1) regular demand and surge demand, 2) variable lead time, 3) two reorder points and two types of orders, and 4) shortage cost. Second, this paper provides a new approach that incorporates LCT and mixed integer programming techniques to obtain the optimal inventory policy. Third, numerical experiments demonstrate the benefit of using the hybrid inventory policy to respond to regular demand and surge demand. Finally this paper extends the application of LCT by considering discrete inventory levels while most of papers in the literature considered the inventory level as continuous states when applying LCT.

The remainder of this paper is organized as follows. A review of the literature related to humanitarian logistics and inventory management facing combined demand patterns is presented in Section 2. The proposed hybrid inventory policy is modeled based on the LCT presented in Section 3. Section 4 formulates a mixed integer program based on the LCT model to optimize the proposed inventory policy. Numerical experiments and sensitivity analyses are conducted in Section 5. Section 6 concludes the paper and provides future research directions.

2. Literature review

Studies on planning and optimizing emergency item inventory management systems to support humanitarian operations have been mostly focused on strategic or operational planning. Inventory management is often considered together with location decisions and stochastic mixed integer programs (SMIP) are often used. Chang et al. [4] formulated two stochastic models for locating warehouses for emergency response in the aftermath of a flood. They developed a solution procedure to solve the flood emergency logistics preparation problems. Facility locations with inventory planning during disaster were studied by Balçik and Beamon [2]. They proposed a stochastic inventory control model that determines optimal order quantities and reorder points for a long-term emergency relief response. Balçik and Beamon [2] assumed that emergency orders are more expensive than regular orders. Similarly, Yu et al. [5] investigated the benefits of dual sourcing under the situation of increased demand during supply disruption. Lodree and Taskin [6] formulated a model for the question of 'appropriate readiness' regarding inventory planning that accommodates demand and extremely uncertain events. They worked within an insurance risk policy framework to find the optimal inventory for efficient disaster relief in the aftermath of a hurricane. Rottkemper et al. [7] presented a quantitative model and a solution approach for planning and optimizing the supply of a specific relief item to a given number of regions after the occurrence of a disruption or a sudden increase of demand. Ozguven and Ozbay [8] performed a case study-based approach to demonstrate the usefulness of a stochastic humanitarian inventory control model and estimation of the minimum safety stock levels of emergency inventories. They present a two-stage SMIP that provides an emergency response pre-positioning strategy for hurricanes or other disaster threats. Rawls and Turnquist [9] presented another two-stage SMIP that provides an emergency response pre-positioning strategy for hurricanes or other disaster threats considering uncertainty in demand for the stocked supplies as well as uncertainty regarding transportation network availability after an event. Campbell and Jones [10] examined the decision of where to preposition supplies in preparation for a disaster and how much to preposition at a location. Considering the supply location risks, they derived equations for determining the optimal stocking quantity and the total expected costs associated with delivering to a demand point from a supply point. A disaster may also cause supply disruption, pre-propositioning emergency inventory at certain supply locations with extra protection used to mitigate the disruption on the supply side, which often happen during large disasters [11]. Increasing the number of suppliers could mitigate supply disruption risk [12,13].

Stochastic inventory control with two types of demand classes has been studied over decades, resulting in a large body of the literature for various settings. Wang et al. [14] studied the rationing policy in an inventory system with two demand classes and different service criteria for backorders with a continuous review (R, Q) system. Kocaga and Alper [15] considered a single-echelon spare parts distribution system with two demand classes and proposed a static rationing policy that would ration stock to non-critical class. Other models studied continuous-review stochastic inventory models with two demand types [16–21]. Inventory control models with two types of demand classes under periodic reviews are studied by [22–27]. The mixture of demand processes has been studied by [28–30]. Azoury et al. [28] considered a continuous review inventory system where demand is a combination of a constant deterministic component and a random component that follows a compound Poisson process. Presman and Sethi [29] showed the optimality of an (s, S)-policy for a continuous-review stochastic inventory model with demand consisting of a compound Poisson process and a constant demand rate. Sobel and Zhang [30] considered a periodic-review inventory system with deterministic and stochastic demands assuming that the deterministic demand has to be satisfied immediately and the stochastic demand can be backordered.

Hybrid inventory policies with two reorder points and two order quantities have been the subject for a number of studies in the literature. Emergency orders may help to reduce backorder [31]. In the context of continuous-review inventory systems Moinzadeh and Nahmias [32] considered a policy with four decision variables, regular reorder level, emergency reorder level, regular order quantity, and emergency order quantity. Under the assumption of full backorder, constant but different lead times for two order types, and at most one outstanding order of each type, they derived an approximate expression for the average cost per unit time and heuristically provided the procedure of determining four decision variables. Mohebbi and Posner [33] developed a model with non-unit-sized demands that also has four policy parameters, regular order quantity, emergency order quantity, regular reorder point, and emergency reorder point. They presented an explicit expression for the average cost rate function under the following assumptions: a compound Poisson demand process, at most one outstanding order of each order type, and exponentially distributed lead time. There are other continuous-review inventory models involving emergency orders closely related to our model [34–38].

Johansen and Thorstenson [34] considered the problem of determining (R, Q) for a single item inventory system in which emergency replenishments are also available in addition to regular inventory replenishment. Axsater [35] derived a new decision rule for emergency orders under compound Poisson demand. Given the reorder point and lot size for regular orders, and based on real-time information regarding the remaining delivery time, he developed a heuristic decision rule to determine the timing and size of the emergency orders. In another study Huang et al. [36] extended the work of Axsater [35] by focusing on an online retailer facing demand that depends on the committed service time and assuming

partial backordering. Kalpakam and Sapna [37] considered two reorder points, two order quantity inventory policies and assumed that a Q-unit order would be placed at regular reorder point R if no order is pending and another order will be placed when the inventory level is zero. The emergency order quantity was assumed to be R. Mamani and Moinzadeh [38] considered a continuous-review inventory system where delivery lead times can be managed by expediting in-transit orders shipped from the supplier. Other studies that focus on periodic-review stochastic inventory models with the regular order mode and expedite order mode are [39–43].

Although the models presented in [34,36] considered emergency orders, their lead time assumptions make them inappropriate for surge demand response. Most of the literature considered two supply modes that are known as regular supply mode and emergency supply mode and assumed that the demand arrival process is either a Poisson process or a compound Poisson process. We consider demand arrival process as a combination of an independent Poisson process and a compound Poisson process. Another difference of our model from the literature is in terms of demand arrivals during lead time. All existing models assumed that emergency orders would occur only during lead time and the inventory level could fall below the emergency reorder point only when there is an outstanding regular order. In our model, as surge demand and regular demand are independent, surge demand can occur at any time and at any inventory position. A surge demand may occur at the maximum inventory position and instantly bring the inventory level to negative if its volume is large. In this case, the inventory level crosses both the regular reorder point and the emergency reorder point at the same moment so that both regular and emergency orders are placed at the same moment. Another issue addressed in our paper is the emergency order quantity. Different from regular order quantity, emergency order quantity depends on the inventory level and volume of surge demand. For example, the size of demand for medicines due to a big earthquake must be bigger than the demand caused by a multiple-car accident.

This paper will apply LCT [44] to obtain the equilibrium distributions of inventory levels under a given inventory policy. Several researchers [33,44–48] have applied LCT in stochastic inventory systems to derive the stationary probability distribution of inventory levels for continuous-review inventory policies. Brill and Chauch [45] used LCT to derive expressions for the distribution and expected value of on-hand inventory, ordering rate, and the expected total cost rate for a given ordering policy. Mohebbi and Posner [33] applied LCT to derive the equilibrium distribution of the inventory level (stock on hand) in a continuous-review inventory system with compound Poisson demand, Erlang as well as hyperexponentially distributed lead times, and lost sales. Mohebbi [46] used LCT to compute the stationary distribution of the on-hand inventory in a continuous-review system facing supply interruptions. For stochastic piecewise linear production and compound Poisson demands, Mohebbi [47] used LCT to derive the stationary distribution of the inventory level in a limited capacity production-storage system with lost sales. Chaouch [48] used LCT to derive the long term inventory distribution function and determine replenishment strategies. He considered the scenario when buyers are faced with price-discounting campaigns that randomly happen. Most of the existing research considered the inventory level as continuous states when applying LCT but our paper considers the inventory level as discrete states.

Applying LCT to obtain equilibrium distribution has advantages over other methods such as the Markov decision process and queuing theory. The state-action space in the underlying semi-Markov decision process is large for the problem addressed in this paper. The queuing theory requires extensive, tedious and time consuming analysis to derive integral equations from the Lindley recursion in order to obtain the probability density function in complex stochastic models with state dependences [44]. LCT is a faster and easier method to derive equations for the probability density. The classical renewal theory is often used to model a variety of continuous-review inventory problems that consider the random demand process as a compound Poisson process with a general distribution for demand sizes [49–52]. Under an assumed order policy, such as (s,S) or (Q,r), those studies relied on the assumption that back orders or lost sales are triggered only during the replenishment lead time. Our paper allows back orders or emergency orders to be triggered at any time points. Therefore, the classical renewal theory is not an option to obtain the equilibrium distribution. Our paper later transforms the LCT model built in Section 3 into a mixed integer program to identify the optimal hybrid policy.

3. Model formulation

Consider a single-location and single-item inventory system at a hospital or humanitarian service provider. The inventory system faces two types of demands, regular demand and surge demand. The regular demand follows a Poisson process at the rate of λ_1 and each requests one unit. The surge demand arrives as a compound Poisson process at the rate of λ_2 and the requested amount of each arrival follows a discrete distribution supported by $[a,b]$. Let r_k be the probability that the requested amount is k, a number between a and b. The assumption that surge demand follows a discrete distribution makes this model more practical for disaster response. Typically, the size of surge demand is highly random and its range could be rather wide. It is also common that surge demand with low demand quantity is more likely to occur as compared to surge demand with high demand quantity. We addressed this issue by developing a discrete distribution expression given by Eq. (44) in Section 5. Because of the nature of the two demand types, we assumed λ_1 is much larger than λ_2 and $a>1$. The inventory policy is defined by four parameters: regular reorder point (R), regular order quantity (Q), emergency reorder point (R_e), and unit emergency order quantity (Q_e). Once the on-hand inventory level reaches or goes below the level of R, a regular order of size Q will be placed and its replenishment lead time is exponentially distributed with a mean of σ^{-1}. An emergency order is placed when the inventory level reaches or goes below the level of R_e. The emergency order quantity is nQ_e, where $R_e<Q_e<R-R_e$ and n is a minimum positive integer value that makes the inventory level go back to a level above R_e. Therefore, $n=\left[(k-i+R_e)/Q_e)+1\right]$, where i is the on-hand inventory just before the surge demand of size k units occurs. Please note that, if the inventory position i is above R when surge demand arrives and the surges demand volume $k\geq i-R_e$, we would place both a regular order and an emergency order. The lead time for the emergency order is assumed to be zero because of the criticality of the medical item.

3.1. LCT modeling framework

LCT is used to obtain the stationary distribution of inventory levels and therefore the long-run average cost per time unit for a given policy featured by a triplet of $(R,\ Q,\ R_e)$. Following Brill [53], we provide the following important definitions.

3.1.1. Definition 1 (state)

Assume that the state space S consists of discrete states $\{w\}$, which represent the inventory levels for this paper. Let $T=\{t|t\in[0,\infty)\}$ and $W(t)$ be the inventory level at time t. For any $w\in S$, $P(W(t)=w)>0$ for some $t\geq 0$ and the steady state probability $\lim_{t\to\infty}P(W(t)=w)>0$, when the limit exists.

3.1.2. Definition 2 (sample path)

A sample path is a bounded real valued or vector valued function $W(t), t \in T$ and $W(t) \in S$. All sample path discontinuities (i.e., the changes that are greater than one unit) are called jumps. During any finite time intervals, the number of jumps and number of relative extrema are finite with finite expectations.

3.1.3. Definition 3 (level)

A level-w in $T \times S$ is defined as a straight line $T \times \{w\}$, $w \in S$.

3.1.4. Definition 4 (down crossing)

Consider $A^0 = (w, \infty) \cap S$ and $A^e = (-\infty, w) \cap S$. $W(\bullet)$ down crosses level w at instant t_0 if $W(\bullet)$ crosses boundary $T \times \{w\}$ at t_0. Equivalently, $W(\bullet)$ exits $T \times A^0$ and enters $T \times A^e$ at t_0.

3.1.5. Definition 5 (up crossing)

$W(\bullet)$ up crosses level w at instant t_0 if $W(\bullet)$ exits $T \times A^e$ and enters $T \times A^0$ at t_0.

LCT starts with constructing sample paths of the underlying stochastic process over time. Although in many applications the construction of sample paths is straightforward, the construction may be nontrivial or challenging in some complex models with high state dependences. For the hybrid inventory policy in this paper, let $\{W(t),\ t > 0\}$ be the inventory level at time t. The inventory level $\{W(t),\ t > 0\}$ at any instant is in the range of $[R_e+1, R+Q]$. Let $P(w) = \lim_{t \to \infty} P(W(t) = w)$, where $w \in [R_e+1, R+Q]$. A sample path in an inventory system may be considered a typical tracing of the inventory level, $W(t)$, over time. Fig. 1 illustrates a sample path for a hybrid inventory policy characterized by $R = 6$, $Q = 9$, $R_e = 2$, and $Q_e = 3$.

The sample path of $W(t)$ in Fig. 1 goes down either by one unit or sometimes by big jumps. The small jumps are caused by regular demands while big jumps are caused by surge demands. The sample path goes up when regular and emergency orders are received. The state space of $W(t)$ can be divided into two subspaces based on whether there is an outstanding regular order. Note that there is no outstanding order when $w \in [R+1,\ R+Q]$, while there is one outstanding regular order when $w \in [R_e+1,\ R]$. These two regions are called page-0 and page-1.

3.2. Development of balancing equations in page-0 and page-1

In page-0, there are two types of down crossings, unit down crossings and jump down crossings. A unit down crossing is caused by a regular unit demand and a jump down crossing occurs when a surge demand arrives. In this page up crossings happen only due to regular order replenishment. By using LCT, we formulate the balance

$$\lambda_1 P(w) + \lambda_2 \sum_{\alpha = w}^{R+Q} \left(\sum_{k = \max(\alpha - w + 1, a)}^{b} r_k \right) P(\alpha) = \sigma \sum_{\alpha = \max(w - Q, R_e + 1)}^{R} P(\alpha) \forall w \in [R+1, R+Q] \tag{1}$$

where $\lambda_1 P(w)$ is the down crossing rate at level w due to regular demands and $\lambda_2 \sum_{\alpha = R+1}^{R+Q} \left(\sum_{k = \alpha - w + 1}^{b} r_k \right) P(\alpha)$ is the down crossing rate due to surge demands. On the right hand side $\sigma \sum_{\alpha = \max(R - Q, R_e + 1)}^{R} P(\alpha)$ is the up crossing rate due to regular order replenishment at level w.

In page-1, the sample path enters into the level of $w \in [R_e+1,\ R]$. The same as page-0, page-1 also has unit down crossings and jump down crossings caused by regular unit demands and surge demand, respectively. In page-1, up crossings occur due to both regular order replenishment and emergency order replenishment. The balance equation can be subdivided into two parts based on the value of w. The balance equation for $w \in [R_e+1\ , R_e+Q_e]$ and $w \in [\ R_e + Q_e + 1, R]$ can be formulated as follows:

$$\lambda_1 P(w) + \lambda_2 \sum_{\alpha = w}^{R+Q} \left(\sum_{k = \max(\alpha - w + 1,\ a)}^{b} r_k \right) P(\alpha) = \sigma \sum_{\alpha = R_e + 1}^{w-1} P(\alpha) + \lambda_2 \sum_{\alpha = R_e + 1}^{R+Q} \left(\sum_{n=1}^{\mathrm{int}((b - \alpha + R_e)/Q_e) + 1} \sum_{k = \max(a, (n-1)Q_e + \alpha - R_e)}^{\min(b,\ nQ_e + \alpha - w)} r_k \right) P(\alpha) + \lambda_1 P(R_e + 1)\ \forall w\ \in [R_e + 1,\ R_e + Q_e] \tag{2.a}$$

$$\lambda_1 P(w) + \lambda_2 \sum_{\alpha = w}^{R+Q} \left(\sum_{k = \max\{\alpha - w + 1, a\}}^{b} r_k \right) P(\alpha) = \sigma \sum_{\alpha = R_e + 1}^{w-1} P(\alpha)\ \forall w\ \in [\ R_e + Q_e + 1, R] \tag{2.b}$$

On the left hand side of both Eqs. (2.a) and (2.b), $\lambda_1 P(w)$ denotes the down crossing rate due to regular demand in page-1, $\lambda_2 \sum_{\alpha = w}^{R+Q} \left(\sum_{k = \max(\alpha - w + 1, a)}^{b} r_k \right) P(\alpha)$ is the jump down crossing rate into interval $w \in (R_e, R+Q]$. On the right hand side of Eqs. (2.a) and (2.b), the term $\sigma \sum_{\alpha = R_e + 1}^{w-1} P(\alpha)$ is the up crossing rate due to regular order replenishment. The term $\lambda_2 \sum_{\alpha = R_e + 1}^{R+Q} \left(\sum_{n=1}^{\mathrm{int}((b - \alpha + R_e/Q_e)) + 1} \sum_{k = \max(a, (n-1)Q_e + \alpha - R_e)}^{\min(b, nQ_e + \alpha - w)} r_k \right) P(\alpha)$ in Eq. (2.a) represents up crossing rate due to emergency order replenishment. At $w = (R_e+1)$, if there is a unit regular demand, it will trigger an emergency order. Therefore, $\lambda_1 P(R_e+1)$ expresses up crossing rate due to an emergency order triggered by regular demand arrival at $(w = R_e+1)$. This emergency order replenishment will result in up crossing to inventory level $w\ \in [R_e+1\ R_e+Q_e]$. So $\lambda_1 P_1(R_e+1)$ is the up crossing rate due to emergency order replenishment when $w\ \in [R_e+1\ R_e+Q_e]$. By adding normalization Eq. (3), we can solve linear Eqs. (1)–(3) to obtain the stationary probability of inventory level w under a given hybrid policy

$$\sum_{w = R_e + 1}^{R+Q} P(w) = 1 \tag{3}$$

3.3. Long-run average cost function under a given hybrid policy

$P(w)$ under a given hybrid policy could be obtained by solving the linear system defined by Eqs. (1)–(3). The inventory system in this paper considers four cost terms: unit inventory holding cost (h), regular order cost (K_1), emergency order cost (K_2), and unit

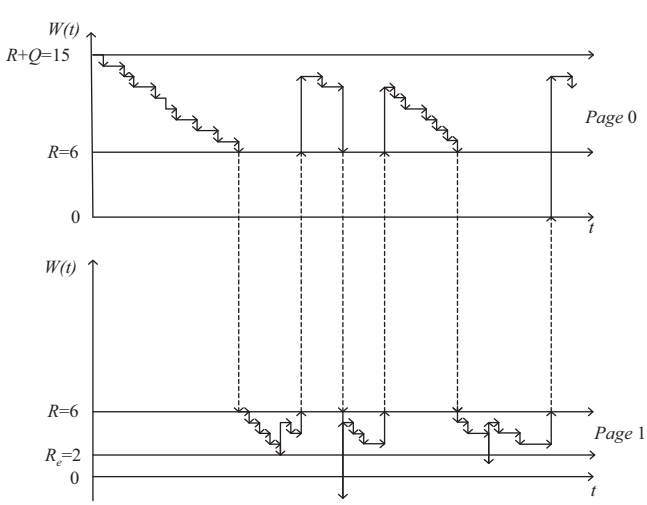

Fig. 1. A sample path of inventory level.

shortage cost (s). For convenience, let $P_w = P(w)$ for $w \in [R_e+1, R+Q]$. The total long-run average cost per time unit, TC(R, Q, R_e, Q_e), can be calculated based on

$$\begin{aligned}\mathrm{TC}(R,Q,R_e,Q_e) = h \sum_{w=R_e+1}^{R+Q} wP_w + \lambda_1 K_1 P_{R+1} \\ + \lambda_2 K_1 \sum_{w=R+1}^{R+Q} \left(\sum_{k=\max(w-R,a)}^{b} r_k \right) P_w \\ + \lambda_1 K_2 P_{R_e+1} + \lambda_2 K_2 \sum_{w=R_e+1}^{R+Q} \left(\sum_{k=\max(w-R_e,a)}^{b} r_k \right) P_w \\ + \lambda_2 S \sum_{w=R_e+1}^{R+Q} \left(\sum_{k=w+1}^{b} (k-w) r_k \right) P_w \end{aligned} \tag{4}$$

where $h\sum_{w=R_e+1}^{R+Q} wP_w$ is the total holding costs, $\lambda_1 K_1 P_{R+1}$ is regular order costs due to regular demands, $\lambda_2 K_1 \sum_{w=R+1}^{R+Q} \left(\sum_{k=\max(w-R,a)}^{b} r_k\right) P_w$ is regular order costs due to surge demands, $\lambda_1 K_2 P_{R_e+1}$ is emergency order costs due to regular demands, $\lambda_2 K_2 \sum_{w=R_e+1}^{R+Q} \left(\sum_{k=\max(w-R_e,a)}^{b} r_k\right) P_w$ is emergency order costs due to surge demands, and $\lambda_2 S \sum_{w=R_e+1}^{R+Q} \left(\sum_{k=w+1}^{b} (k-w) r_k\right) P_w$ is total shortage costs.

4. Mixed integer program for optimal policy

An optimization model with Eq. (4) as the objective function and Eqs. (1)–(3) as constraints could be used to obtain the optimal policy. However, the model with Eqs. (1)–(4) is not a linear program with embedded nonlinear functions of int( ), max( ), and min( ). One possible method could be applying total enumeration to find an optimal policy. By solving Eqs. (1)–(3)) for any given R, Q, Q_e and R_e we can obtain P_w and then calculate the total long-run average cost per time unit by using Eq. (4). However, total enumeration is not computationally efficient. A mixed integer programming (MIP) model is developed in this section to obtain an optimal policy (R^*, Q^*, R_e^*) by linearizing balance Eqs. (1), (2.a) and (2.b) and the objective function in Eq. (4). The MIP assumes that $R+Q-R_e < U$, where U is an upper bound of $R+Q$. The MIP also assumes that Q_e is given. In practice, Q_e is often the minimum measurement unit and is fixed. The linearization requires a number of additional variables and constrains, which are defined as follows.

$y_{w\alpha}$: probability of down crossings of level w from level α due to surge demand arrival;
$t_{w\alpha}$: probability of up crossings of level w from level α due to regular order replenishment;
$m_{w\alpha k}$: for a given $w \in [R_e+1, R_e+Q_e]$ and $\alpha \in [R_e+1, R+Q]$ the probability of surge demand k for which an up crossing of level w from level α occurs due to emergency order arrival;
$n_{\alpha k}$: number of emergency orders of size Q_e required to keep the inventory level above R_e when a surge demand of size k occurs at inventory level α;
$P_w^1 =$ P_{R_e+1} if $w = R_e+1, 0$: otherwise;
$P^2 =$ P_{R+1};
$v_{wk}^1 (v_{wk}^2)$: probability for surge demand of size k to trigger a regular order (emergency orders) at level w;
M: a big number;
$Z_w^1 =$ 1 : if $w \le R+Q, 0$: otherwise;
$Z_w^2 =$ 1 : if $w \ge R_e+1, 0$: otherwise;
$Z_w^3 =$ 1 : if $w \le R, 0$: otherwise;
$Z_w^4 =$ 1 : if $w \le R_e+Q_e, 0$: otherwise;
$Z_w^5 =$ 0 : if $w \le R+1, 1$: otherwise;
$Z_w^6 =$ 0 : if $w \le R_e+1, 1$: otherwise;
$B_{w\alpha} =$ 1 : if $\alpha \ge w-Q, 0$: otherwise for $w \in [1, U], \alpha \in [1, U]$;
$N_{wk} =$ 1 : if $k \ge w-R_e, 0$: otherwise for $w \in [1, U], k \in [a, b]$;
$L_{wk} =$ 1 : if $k \ge w-R, 0$: otherwise for $w \in [1, U], k \in [a, b]$; and
$J_{w\alpha k} =$ 1 : if $\alpha - k + n_{\alpha k} Q_e \ge w, 0$: otherwise for $w \in [1, U]$, $\alpha \in [1, U], k \in [a, b]$.

Introducing P_w^1, P^2, v_{wk}^1 and v_{wk}^2, we can rewrite the objective function in Eq. (4) into Eq. (5) as the objective function for the MIP

$$\begin{aligned}\text{Min } TC(R,Q,R_e,Q_e) = h \sum_{w=1}^{U} wP_w + \lambda_1 K_1 P_{R+1} \\ + \lambda_2 K_1 \sum_{w=1}^{U} \sum_{k=a}^{b} v_{wk}^1 + \lambda_1 K_2 \sum_{w=1}^{U} wP_w^1 \\ + \lambda_2 K_2 \sum_{w=1}^{U} \sum_{k=a}^{b} v_{wk}^2 \\ + \lambda_2 S \sum_{w=1}^{U} \left(\sum_{k=w+1}^{b} (k-w) r_k \right) P_w. \end{aligned} \tag{5}$$

By introducing variables $P_w, y_{w\alpha}$, $t_{\alpha w}$ and P_w^1 we combine Eqs. (1), (2.a) and (2.b) into

$$\lambda_1 P_w + \lambda_2 \sum_{\alpha=w}^{U} y_{w\alpha} = \sigma \sum_{\alpha=1}^{w-1} t_{w\alpha} + \lambda_2 \sum_{\alpha=1}^{U} \sum_{k=a}^{b} m_{w\alpha k} + \lambda_1 P_w^1 \quad \forall w \in [1, U]. \tag{6}$$

The left hand side of constraint in Eq. (6) is the total down crossing rate over the level of $w \in [1, U]$, including those caused by the regular unit demand ($\lambda_1 P_w$) and the jump down crossing rate from $\alpha \in [w, U]$. The right hand side is the total possible up crossing rate for all $w \in [1, U]$. Left hand side variables P_w and $y_{w\alpha}$ in Eq. (6) take positive values only when $w \in [R_e+1, R+Q]$, which will be forced by additional constraints in Eqs. (7)–(9) with the new binary variables of Z_w^1 and Z_w^2, whose values are determined by the inventory policy (R, Q, R_e). Constraint sets in Eqs. (7) and (8) set $Z_w^1 = 1$ if $w \le R+Q$ and $Z_w^1 = 0$, otherwise and $Z_w^2 = 1$ if $w \ge R_e+1$. Constraint set in Eq. (9) forces P_w to be 0 if $w \le R_e$ or $w > R_e$. Constraint sets in Eqs. (10)–(12) make sure that $y_{w\alpha} = \sum_{k=\max(\alpha-w+1,a)}^{b} r_k P_\alpha$ when $\alpha, w \in [R_e+1, R+Q]$. Please note that $\alpha \ge w$ here

$$R+Q-w+1 \le MZ_w^1 \le M-w+R+Q \quad \forall w \in [1, U]; \tag{7}$$

$$w-R_e \le MZ_w^2 \le M-R_e-1+w \quad \forall w \in [1, U]; \tag{8}$$

$$P_w \le Z_w^1 + Z_w^2 - 1 \quad \forall w \in [1, U]; \tag{9}$$

$$\begin{aligned}\sum_{k=\max(\alpha-w+1,a)}^{b} r_k P_\alpha - 3 + Z_\alpha^1 + Z_w^1 + Z_w^2 \le y_{w\alpha} \le \sum_{k=\max(\alpha-w+1,a)}^{b} r_k P_\alpha \\ + 3 - Z_\alpha^1 - Z_w^1 - Z_w^2 \quad \forall w \in [1, U], \alpha \in [w, U]; \end{aligned} \tag{10}$$

$$y_{w\alpha} \le P_\alpha \quad \forall w \in [1, U], \alpha \in [w, U]; \text{ and} \tag{11}$$

$$y_{w\alpha} \le Z_w^2 \quad \forall w \in [1, U], \alpha \in [w, U]. \tag{12}$$

For the right hand side of Eq. (6), constraint sets in Eqs. (13)–(19) are used to get $t_{\alpha w} = P_\alpha$ only when $\alpha \in [R_e+1, R], w \in [R_e+1, R+Q]$ and $\alpha \ge w-Q$. Here, $Z_w^3 = 1$ if $w \le R; Z_w^3 = 0$, otherwise. $B_{w\alpha} = 1$ if $\alpha \ge w-Q; B_{w\alpha} = 0$, otherwise

$$\begin{aligned}P_\alpha - 5 + (Z_w^1 + Z_w^2 + Z_\alpha^2 + Z_\alpha^3 + B_{w\alpha}) \le t_{w\alpha} \le P_\alpha \\ + 5 - (Z_w^1 + Z_w^2 + Z_\alpha^2 + Z_\alpha^3 + B_{w\alpha}) \quad \forall w \in [1, U], \alpha \in [1, w-1]; \end{aligned} \tag{13}$$

$$t_{w\alpha} \le Z_w^1 \quad \forall w \in [1, U], \alpha \in [1, w-1]; \tag{14}$$

$$t_{w\alpha} \le Z_\alpha^2 \quad \forall w \in [1, U], \alpha \in [1, w-1]; \tag{15}$$

$$t_{w\alpha} \le Z^3_\alpha \quad \forall w \in [1,\ U], \alpha \in [1,\ w-1]; \tag{16}$$

$$t_{w\alpha} \le MB_{w\alpha} \quad \forall w \in [1,\ U], \alpha \in [1,\ w-1]; \tag{17}$$

$$R-w+1 \le MZ^3_w \le M-w+R \quad \forall w \in [1, U]; \text{ and} \tag{18}$$

$$\alpha-w+Q+1 \le MB_{w\alpha} \le M+\alpha-w+Q \quad \forall w \in [1, U], \alpha \in [1,\ w-1]. \tag{19}$$

Constraint sets in Eqs. (20)–(30) are used to set $m_{w\alpha k} = r_k P_\alpha$ only when $w \in [R_e+1,\ R_e+Q_e], \alpha \in [R_e+1\ ,\ R+Q],\ k \ge \alpha - R_e$ and $\alpha - k + n_{\alpha k} Q_e \ge w$ and enforcing $m_{w\alpha k}$ to be zero otherwise. Here, $m_{w\alpha k}$ is the probability that a surge demand of size k causes an up crossing of level w from level α due to emergency orders

$$r_k P_\alpha - 6 + (N_{\alpha k} + Z^2_\alpha + Z^2_w + Z^4_w + Z^1_\alpha + J_{w\alpha k}) \le m_{w\alpha k} \le r_k P_\alpha + 6 - (N_{\alpha k} + Z^2_\alpha + Z^2_w + Z^4_w + Z^1_\alpha + J_{w\alpha k}) \quad \forall w \in [1, U],\ \alpha \in [1, U], k \in [a, b]; \tag{20}$$

$$m_{w\alpha k} \le P_\alpha \quad \forall w \in [1, U],\ \alpha \in [1, U], k \in [a, b]; \tag{21}$$

$$m_{w\alpha k} \le Z^2_w \quad \forall w \in [1, U],\ \alpha \in [1, U], k \in [a, b]; \tag{22}$$

$$m_{w\alpha k} \le Z^4_w \quad \forall w \in [1, U],\ \alpha \in [1, U], k \in [a, b]; \tag{23}$$

$$m_{w\alpha k} \le J_{w\alpha k} \quad \forall w \in [1, U],\ \alpha \in [1, U], k \in [a, b]; \tag{24}$$

$$m_{w\alpha k} \le N_{\alpha k} \quad \forall w \in [1, U],\ \alpha \in [1, U], k \in [a, b]; \tag{25}$$

$$\alpha - k + n_{\alpha k} Q_e - w + 1 \le MJ_{w\alpha k} \le M + \alpha - k + n_{\alpha k} Q_e - w \quad \forall w \in [1, U],\ \alpha \in [1, U], k \in [a, b]; \tag{26}$$

$$R_e + 1 - M(1 - N_{\alpha k}) \le \alpha - k + n_{\alpha k} Q_e \le R_e + Q_e + M(1 - N_{\alpha k}) \quad \forall w \in [1, U],\ \alpha \in [1, U], k \in [a, b]; \tag{27}$$

$$n_{\alpha k} \le MN_{\alpha k} \quad \forall \alpha \in [1, U], k \in [a, b]; \tag{28}$$

$$R_e - \alpha + k + 1 \le MN_{\alpha k} \le M - \alpha + k + R_e \quad \forall \alpha \in [1, U], k \in [a, b]; \text{ and} \tag{29}$$

$$R_e + Q_e - w + 1 \le MZ^4_w \le M - w + R_e + Q_e \quad \forall w \in [1, U]; \tag{30}$$

Following the same logic, linear constraint sets in Eqs. (31)–(35) are used to obtain P^1_w and P^2, whose values depend on R and R_e. P^1_w and P^2 are used in constrain set in Eq. (6) and objective function in Eq. (5)

$$P_w + Z^2_w - Z^6_w - 1 \le P^1_w \le P_w + 1 - Z^2_w + Z^6_w \quad \forall w \in [1, U]; \tag{31}$$

$$P^1_w \le Z^2_w - Z^6_w \quad \forall w \in [1, U]; \tag{32}$$

$$w - R_e - 1 \le MZ^6_w \le M + w - R_e - 2 \quad \forall w \in [1, U]; \tag{33}$$

$$P_w - (Z^3_w + Z^5_w) \le P^2 \le P_w + (Z^3_w + Z^5_w) \quad \forall w \in [1, U]; \text{ and} \tag{34}$$

$$w - R - 1 \le MZ^5_w \le M + w - R - 2 \quad \forall w \in [1, U]. \tag{35}$$

The following constraints set in Eqs. (36)–(40) are used to obtain v^1_{wk}, the probability that a surge demand of size k triggers a regular order at level w. Similarly, v^2_{wk} is the probability that a surge demand of size k triggers an emergency orders and is determined by constraint sets in Eqs. (41) and (42)

$$r_k P_w - 1 + L_{wk} - Z^3_w \le v^1_{wk} \le r_k P_w + 1 - L_{wk} + Z^3_w \quad \forall w \in [1, U],\ k \in [a, b]; \tag{36}$$

$$v^1_{wk} \le 1 - Z^3_w \quad \forall w \in [1, U],\ k \in [a, b]; \tag{38}$$

$$v^1_{wk} \le L_{wk} \quad \forall w \in [1, U],\ k \in [a, b]; \tag{39}$$

$$k - w + R + 1 \le ML_{wk} \le M + k - w + R \quad \forall w \in [1, U],\ k \in [a, b]; \tag{40}$$

$$r_k P_w - 1 + N_{wk} \le v^2_{wk} \le r_k P_w + 1 - N_{wk} \quad \forall w \in [1, U],\ k \in [a, b]; \text{ and} \tag{41}$$

$$v^2_{wk} \le N_{wk} \quad \forall w \in [1, U],\ k \in [a, b]. \tag{42}$$

Replacing $R+Q$ by U and R_e+1 by 1, the normalization Eq. (3) can be rewritten as

$$\sum_{w=1}^{U} P_w = 1. \tag{43}$$

The MIP with objective function in Eq. (5) and constraints in Eqs. (6)–(43) can be solved easily using a commercial optimization package, such as IBM ILOG CPLEX.

5. Numerical results and sensitivity analysis

Numerical experiments are conducted based on the MIP model in Eqs. (5)–(43) developed in Section 4 to gain insights on the optimal hybrid inventory policy responding to both regular and surge demands. In the literature there are no benchmark data sets that we could directly use to test this particular problem and evaluate the applicability of the model and solution approach. Therefore, the data sets used in the experiments are randomly generated with demand distributions that could reflect real applications. The results also demonstrate the effect of the hybrid inventory policy compared to the inventory policy without emergency order.

5.1. Experimental design

The optimization model was solved by IBM CPLEX 12.5.1 using the Concert Technology in C++. All tests were conducted using a desktop computer with Intel ® Core i7 3.1 GHz CPU and 32 GB memory. The optimality tolerances were all set to 1%. Various scenarios with different input parameters were used to test the MIP model.

The surge demand size is assumed to follow the discrete distribution defined by Eq. (44) such that r_k decreases in k with a constant rate. In practice, small incidents such as car accidents are more likely to occur than big accidents such as nuclear accidents. However, any other discrete distributions could be used to model the surge demand distribution r_k without significantly increasing computational burden and will be discussed in Section 5.3

$$r_k = \frac{2(b-k+1)}{(b-a+2)(b-a+1)}, \text{ where } k \in [a\ ,\ b]. \tag{44}$$

5.2. Computational results

Table 1 presents the optimal policy (R, Q, R_e), minimum cost, and computational time for various instances. We can see that the computational time increases as the value of U (the maximum possible inventory level) increases. The results show that commercial solvers can solve the proposed MIP model within thousands of seconds when U is 60 or smaller. Computational time could be reduced by selecting an appropriate value of U. A high value of U increases the number of binary variables and the number of constraints in the MIP and therefore may cause a computational challenge. In this case, further study on algorithms and possibly approximations will be necessary to address the computational burden. In general, the value of U depends on the estimated value of $R+Q$ for the optimal inventory policy. In practice, it is possible to roughly pre-estimate the values of Q based on the Economic Order Quantity (EOQ) to balance the inventory holding cost and the fixed cost of regular orders.

Table 1
R, Q, R_e, optimal cost and computing time for different instances.

λ_1	λ_2	σ	U	h	K_1	K_2	a	b	Q_e	s	R^*	Q^*	R_e^*	Optimal cost	Com. time (s)
1	0.01	1	40	0.8	40	200	2	30	3	150	6	16	0	18.27	601
				1	25	140	2	50	3	80	4	12	0	21.57	609
1.5	0.02	1.5	40	0.8	40	200	2	30	3	150	9	20	0	22.92	626
				1	25	140	2	50	3	80	7	19	0	29.99	620
2	0.025	2	50	0.8	40	200	2	30	3	150	10	22	0	25.03	1698
				1	25	140	2	50	3	80	8	23	0	33.35	1701
2.5	0.03	2	50	0.8	40	200	2	30	3	150	11	23	0	26.72	1750
				1	25	140	2	50	3	80	10	24	0	35.95	1769
3	0.035	3	60	0.8	40	200	2	30	3	150	12	24	0	28.47	4021
				1	25	140	2	50	3	80	12	25	0	38.39	4140
3.5	0.04	3	60	0.8	40	200	2	30	3	150	12	26	0	30.08	4289
				1	25	140	2	50	3	80	14	26	0	40.53	4317
4	0.045	3.5	60	0.8	40	200	2	30	3	150	13	27	0	31.39	4319
				1	25	140	2	50	3	80	15	27	0	42.26	4410
4.5	0.05	4	60	0.8	40	200	2	30	3	150	13	28	0	32.62	4403
				1	25	140	2	50	3	80	16	28	0	43.81	4420
5	0.055	4.5	60	0.8	40	200	2	30	3	150	14	29	0	33.77	4397
				1	25	140	2	50	3	80	17	29	0	45.23	4430
5.5	0.06	5	60	0.8	40	200	2	30	3	150	14	30	0	34.85	4519
				1	25	140	2	50	3	80	18	29	0	46.54	4850

Table 2
Percentage of cost savings due to the hybrid inventory policy.

λ_1	λ_2	σ	h	K_1	K_2	a	b	Q_e	s	Basic Inventory Policy			Hybrid Inventory Policy				Percent of cost savings
										R^*	Q^*	Optimal cost	R^*	Q^*	R_e^*	Optimal cost	
1	0.2	1	0.8	25	140	2	30	3	1000	44	15	49.83	39	14	18	45.03	9.62
1	0.2	1	0.8	25	140	2	40	3	1500	64	14	67.00	43	30	29	57.35	14.41
1.5	0.2	1.5	0.8	25	140	2	40	3	1000	53	17	59.31	41	30	25	55.63	6.19
2	0.2	2	0.8	25	140	2	40	3	1500	52	20	59.93	41	31	26	56.44	5.83
3	0.2	3	0.8	25	140	2	40	3	1000	53	24	62.29	49	24	22	58.93	5.39
3.5	0.3	3.5	0.8	25	140	2	40	3	1500	52	27	63.98	45	34	24	61.54	3.82
4	0.3	3	0.8	25	140	2	40	3	1500	54	28	66.33	47	34	24	63.15	4.80
4.5	0.3	3.5	0.8	25	140	2	40	3	2000	54	30	67.08	47	35	25	63.77	4.93
4.5	0.3	3.5	0.8	25	140	2	40	3	3000	57	30	69.70	48	35	28	64.90	6.89
5	0.4	4	0.8	25	140	2	50	3	4000	79	35	92.02	62	41	38	80.95	12.04

Its values is mainly influenced by K_1, λ_1, and h but could also be influenced by surge demand arrival rate λ_2 and the expected surge demand size. Regular reorder point R mainly depends on the lead time demand (i.e., λ_1 and σ^{-1}) to balance inventory holding cost and shortage cost. However, we suspect that a high unit shortage cost (s) may not cause a big increase of regular reorder point because of possible emergence orders. All above discussions are supported by the following numerical experiment results.

Additional instances were generated to investigate the benefit of using the hybrid inventory model over the basic inventory policy without emergency orders. A model for the basic inventory policy was built by simply removing the up crossing rates of $\lambda_2 \sum_{\alpha=R_e+1}^{R+Q} \left(\sum_{n=1}^{\mathrm{int}((b-\alpha+R_e)/Q_e)+1} \sum_{k=\max(a,(n-1)Q_e+\alpha-R_e)}^{\min(b,\ nQ_e+\alpha-w)} r_k \right) P(\alpha) + \lambda_1 P(R_e+1)$ from the right side of Eq. (2.a) in the LCT balancing equations and by modifying the corresponding MIP. After removing these up crossing rates caused by emergency orders, Eqs. (2.a) and (2.b) are combined into one equation and both R_e and Q_e disappear after we replace R_e with a lower bound of the inventory level in Eqs. (1)–(3). The percentages of savings obtained by the hybrid inventory policy are shown in Table 2. The percentage of savings due to the hybrid inventory model varies from 3.82% to 14.41% across instances.

5.3. Sensitivity analysis

To investigate the impact that the values selected as problem parameters have on costs and replenishment policy, we conducted a sensitivity analysis with results presented in Figs. 2–7 with discussions. The parameters used in the base case for the sensitivity analysis are presented in Table 3.

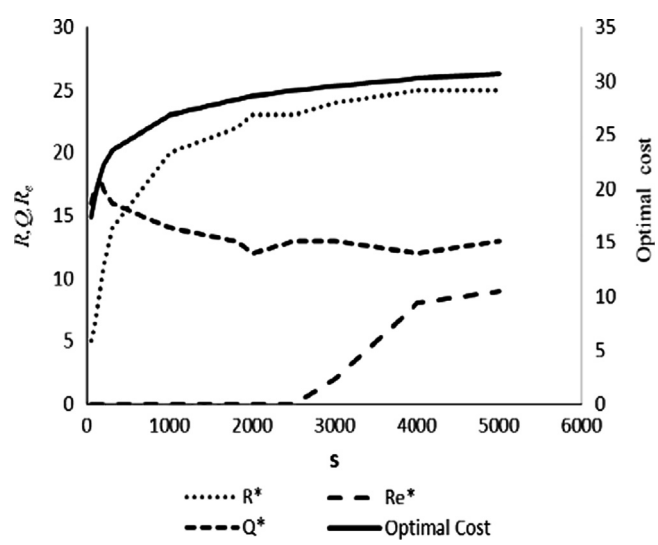

Fig. 2. Sensitivity analysis of s.

Observation 1: Similar to the traditional (Q, r) inventory model, Fig. 2 indicates that the optimal cost and regular reorder

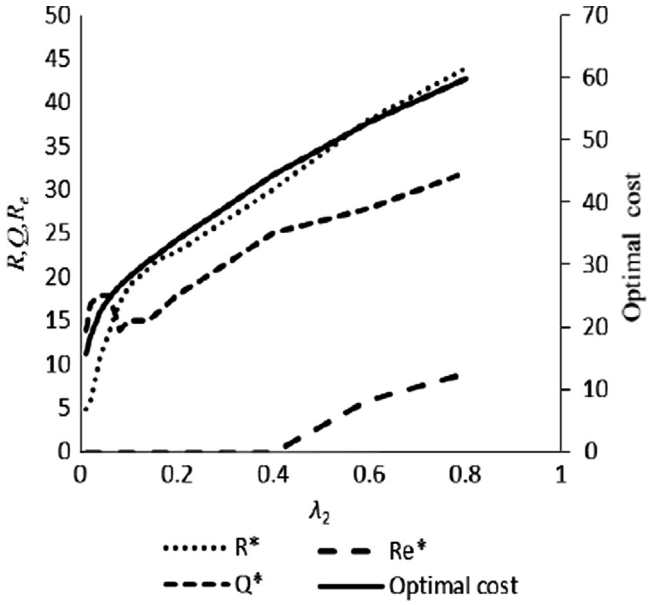

Fig. 3. Sensitivity analysis of λ_2.

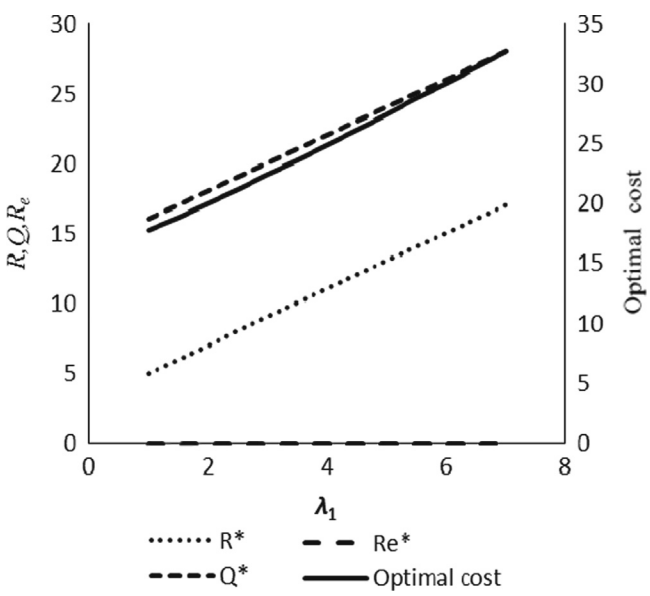

Fig. 4. Sensitivity analysis of λ_1.

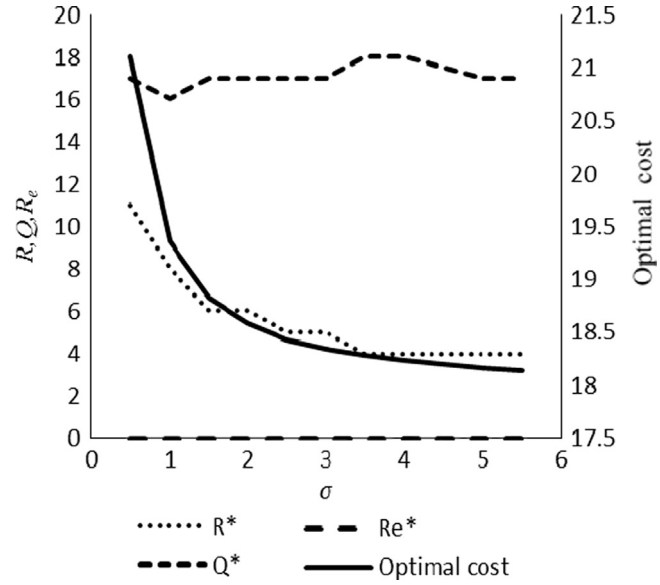

Fig. 5. Sensitivity analysis of σ.

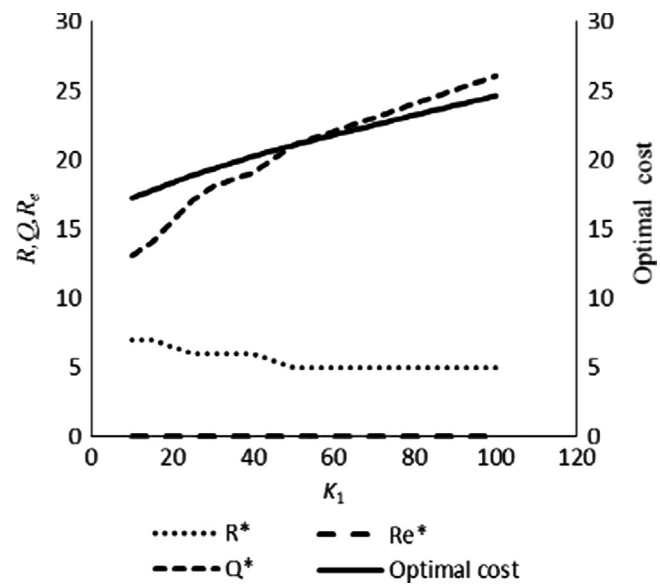

Fig. 6. Impact of K_1.

point R quickly increase with the unit shortage cost s, especially when s is small (e.g., $s<1500$). However, R becomes relatively stable later when s is large (e.g., $s>2500$) because the shortage cost is controlled mainly through a positive and increasing R_e in the case with high s rather than through increasing the maximum inventory level. The regular order quantity Q in general keeps the same with a slight decrease because the regular reorder points goes up. The value of Q mainly depends on the fixed ordering cost and unit inventory holding cost, similar to the traditional Economic Order Quantity (EOQ). Because there are multiple decision variables and the optimality gap (i.e., 1% gap), the monotonicity does not strictly hold. For example, in Fig. 2, when s is about at 2000, the regular order quantity Q has a drop while the regular reordering point has a bump. Similar phenomenon exists in other figures. The emergency reorder point R_e is sensitive to both unit shortage cost s and surge demand arrival rate λ_2 (see Figs. 2 and 3). If the unit shortage cost is low, then $R_e=0$, implying that the emergency orders only happen when the inventory is used up. Fig. 3 shows that the optimal cost and the values of R, Q, and R_e all increase in λ_2. All of them are intuitive. Higher λ_2 implies higher overall demand rate so that the regular order quantity goes up. Higher λ_2 also implies higher expected demand during the regular order lead time so that the regular reorder point R goes up. When λ_2 is high enough, the emergency reorder point R_e could become non-zero, which means emergency orders are placed to prepare for possible surge demand even when the inventory is not used up. These results imply that a good practice for an inventory management team for responding to surge demand is to categorize inventories based on their unit shortage cost s (i.e., criticality) and surge demand frequency λ_2 and then to decide to set the emergency reordering point at zero or a positive value.

Observation 2: Fig. 4 shows that the regular demand rate influences the regular order quantity Q and the regular reorder point R similar to a traditional (Q,r) inventory system, in which higher demand rate causes bigger order quantity Q and higher regular reorder point R. The decision about setting the value of R_e to positive depends on the ratio of λ_2/λ_1. When this ratio is small,

$R_e = 0$, (see Figs. 3 and 4). As a result, items required to respond to car accidents, a kind of disaster that occurs frequently (i.e., higher λ_2), tend to have higher regular reorder point, higher emergency reorder point, and larger order quantity.

Observation 3: Fig. 5 shows the impact of expected lead time σ^{-1} of regular orders on the optimal cost and values of R, Q, and R_e. Larger σ means smaller expected value and variance of the regular order lead time, which is assumed to follow an exponential distribution. Obviously longer regular order lead time σ^{-1} increases the regular reorder point R but has little impact on the regular order quantity Q and emergency reorder point R_e.

Observation 4: The regular order quantity Q is sensitive to K_1 (as shown in Fig. 6). As K_1 increases, Q increases and R decreases. This observation is similar to the regular inventory management without emergency orders. K_1 does not have any impact on R_e, which remains zero. Fig. 7 indicates that R rather than R_e is more sensitive to K_2. Moreover, from Figs. 6 and 7 we can see that K_1 and K_2 do not have any impact on R_e. When unit emergency order cost K_2 is higher, the overall inventory level becomes higher with higher regular reorder R to avoid frequent emergency orders. We expect that very small K_2 may trigger a non-zero emergency reorder point. However, K_2 is typically much higher than K_1 in reality in order to have emergency orders immediately or almost immediately for critical healthcare and humanitarian items.

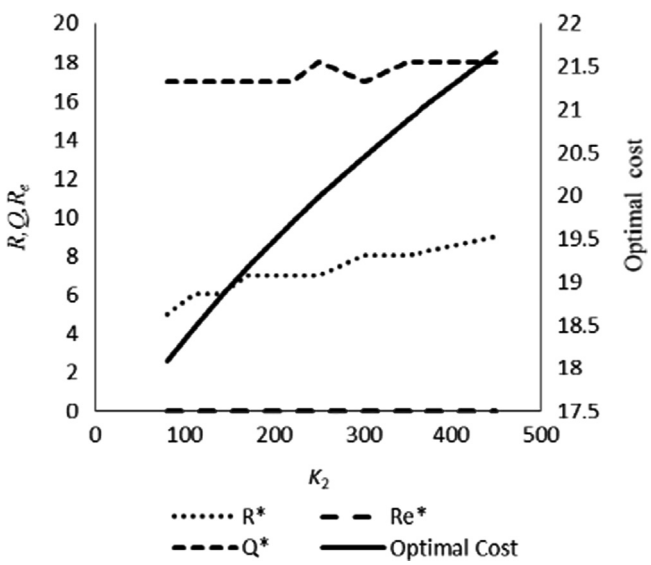

Fig. 7. Impact of K_2.

Table 3
Base line parameter values for numerical experiments.

λ_1	λ_2	σ	h	K_1	K_2	a	b	Q_e	s
1.5	0.02	1.5	0.8	25	140	2	30	3	80

Observation 5: Experimental results indicate that R_e is zero in most of the cases (Figs. 4–7). This may be due to the fact that our emergency order lead time is zero. This assumption tends to avoid maintaining additional safety stock. However, if the shortage cost s and/or surge demand arrival rate λ_2 are high, R_e becomes positive as shown in Figs. 2 and 3.

Observation 6: A sensitivity analysis of the distribution of the surge demand size was also conducted. Table 4 presents a comparison among distributions of a decreasing density function as in Eq. (44), a uniform distribution, and a more general triangular distribution with a different mode parameter c, where the probability density reaches the maximum. Without a surprise, both the optimal safety stock level and total cost increase when the peak of the density function shifts to the upper bound of the distribution range, b, because the shift means higher overall demand rate. The computational time under different distributions could be different because of the different values of U, which needs to be larger than R^*+Q^*. However, we do not expect the difference is huge as long as the range of the distribution $[a, b]$ keeps the same.

Observation 7: Figs. 2–7 show that $R_e = 0$ works well in most of the settings, which is similar to the observation in the literature that emergency orders are usually triggered by zero or negative inventory levels [33–35]. Since this paper assumes zero lead time for emergency orders, same as [35], and does not allow backorders due to the importance of the item, negative R_e is not allowed. Numerical results in [34] show that in most cases R_e is zero or negative when backorders are allowed.

6. Conclusion and future research

This paper develops a stochastic inventory model based on a hybrid inventory policy with both regular and emergency orders responding to regular and surge demands. LCT is applied in order to obtain the equilibrium distributions of inventory levels under a given policy. Different from most papers in the literature that applied LCT to the cases with exponentially distributed demand size, this paper assumes that demand volume follows a discrete distribution. To the best of our knowledge, this paper is the first effort of applying LCT to the cases with a discrete state space. This paper further transforms the model into a mixed integer program to identify the optimal hybrid policy. The numerical experiments show that commercial solvers can solve the proposed MIP model within thousands of seconds when U (the maximum possible

Table 4
Comparison of optimal R, Q, R_e and cost for different demand distributions of surge demand.

Base: $r_k = \frac{2(b-k+1)}{(b-a+2)(b-a+1)} \forall k \in [a, b]$				Uniform: $r_k = \frac{1}{b-a} \forall k \in [a, b]$				Triangular: $r_k = \begin{cases} \frac{2(k-a)}{(b-a)(c-a)} & \forall a \le k \le c \\ \frac{2(b-k)}{(b-a)(b-c)} & \forall c \le k \le b \end{cases}$				
R^*	Q^*	R_e^*	Optimal cost	R^*	Q^*	R_e^*	Optimal cost	c	R^*	Q^*	R_e^*	Optimal cost
								3	7	17	0	19.19
								6	7	18	0	19.79
								9	8	18	0	20.46
								12	8	19	0	21.19
6	17	0	18.82	9	21	0	23.89	15	9	19	0	21.97
								18	10	19	0	22.81
								21	11	19	0	23.69
								24	12	19	0	24.61

inventory level) is moderate, such as 60 or smaller. It is expected that a very large U could lead to a computational challenge and require further study on algorithm development and possibly some approximations. This model could help healthcare providers or humanitarian logistics providers in managing their emergency supplies in responding to surge demands for certain items. In order to apply the solution approach in the real world, the surge demand arrival rate and the distribution of its size per arrival need to be estimated. The estimation could be a challenge because there are often no enough historical data for surge demand compared to regular demand.

Numerical results clearly show the benefit of using this hybrid inventory model with 3.82–14.41% total cost savings. Furthermore, a sensitivity analysis is conducted in the numerical experiments to show how different parameter values affect the optimal inventory policy and its cost. Most observations are intuitive and similar to the traditional (Q, r) inventory model. However, it is interesting to see that the emergency reorder point becomes positive when the impact of a shortage is large, which is often the case for humanitarian supply chain. A positive emergency reorder point could be a good mechanism to reduce the overall inventory level for critical items whose shortage may lead to a big loss. It is also interesting to see that higher emergency order cost will drive the regular order point higher if the item is critical and an emergency order must be placed when there is a shortage.

This paper only considers a single-location, single-item inventory system. The model could be extended to multiple locations. However, multiple-locations will not only increase the size of the MIP problem but also introduce the decision of demand assignment among locations. Due to the large problem size, commercial optimization solvers may not be able to solve the problems in a reasonable amount of time. Therefore, decomposition or heuristic algorithms may be necessary to obtain quality hybrid inventory policies for multiple-location cases. During natural disasters and man-made events, multiple items may be needed. When the items have substitute or complementary relationship, their inventory policies need to be considered in a systematic way. However, a multi-item model with emergency orders is expected to be much more complicated and may not be computationally tractable. Furthermore, this paper assumes that items are not perishable though many medicines have finite shelf life in practice. One future effort could be to incorporate perishability into the model. We expect that emergency orders may help to address the perishability issue, which often causes a conflict between shortage and expiry.

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